

Math 477, Homework 9

Name: _____

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1. Suppose that a fair die is rolled n times. We say that there is a repeat at the i 'th place if the same number occurs on both the i 'th and $i + 1$ 'st roll. Let X be a random variable representing the number of repeats. Find $E[X]$.
2. Suppose that a fair die is rolled n times. We say that there is an increase at the i 'th place if result on the $i + 1$ 'st roll is greater than the result on the i 'th roll. Let X be a random variable representing the number of increases. Find $E[X]$.
3. Let X and Y be independent random variables, uniformly distributed on the interval $[0, 2]$. Find $E[e^{X+Y}]$.
4. Suppose that in a certain game you roll a die, and get winnings equal to \$100 times the amount shown on the die. If you want to, you can roll again up to a total of three times. However, each time you roll again, you forfeit your previous winnings.
You decide to take the following strategy: choose a number i , and if you ever get i or above, stop and collect your winnings, otherwise roll again. What is the value of i which maximizes your expected winnings?
5. Suppose that you have a bucket filled with coins of different types. We say that a coin is type p for $p \in [0, 1]$ if the coin comes up heads with probability p . Let P be the random variable representing the type of coin drawn, when one is drawn at random. Suppose that P has probability density function $f(x) = 2x$ for $x \in [0, 1]$.
Let X be the number of heads obtained after 5 rolls from a coin chosen randomly from the bucket. Find $E[X]$.