

Math 477, Practice sheet for Exam 1

This exam will cover sections 1.1–1.6, 2.1–2.5, 3.1–3.5, 4.1–4.7 and 4.8.1, 4.8.2.

The problems below do not necessarily cover all the topics completely, but will hopefully still be helpful as a reminder of some of the material covered. Please also look through prior worksheets and homework assignments.

1. Consider sequences of n numbers, each in the set $\{1, 2, \dots, 6\}$.
 - (a) How many sequences are there if each number in the sequence is distinct?
 - (b) How many sequences are there if no two consecutive numbers are equal?
 - (c) How many sequences are there if 1 appears exactly i times in the sequence?
2. Suppose that a basket contains n red balls and m blue balls. The balls are removed from the basket, each one equally likely until r red balls have been removed. What is the probability that a total of k balls have been removed at this point?
3. Suppose that we have 2 coins, the first, when flipped, has a 90% heads and the second has a 50% chance of resulting in heads or tails. Suppose a coin is picked (each coin being equally likely). When flipped, the result is heads. What is the probability that, if flipped again, the result will be heads?
4. Suppose that we have 2 coins, the first, when flipped, has a 90% heads and the second has a 50% chance of resulting in heads or tails. Suppose a coin is picked (each coin being equally likely), and is flipped over and over until the result is tails. What is the expected number of flips this will take?
5. Two dice are rolled, and the results are added. Assuming that this number is greater than or equal to 8, what is the probability that one of the dice rolled a 6?
6. In the game “raven’s beak,” a player rolls 6 dice, and wins if at least three of the dice roll the same number. What is the probability of winning?
7. In the game “dove’s gambit,” a player rolls 6 dice, and wins if at least three of the dice roll the number 1. What is the expected number of games played before the player wins?
8. A swarm of flying insects are flying around a lamp. If there are 1000 insects, and each insect has a probability of $1/2000$ of bumping into the light every second, estimate the probability that no more than 3 insects hit the lamp after 10 seconds?
9. A swarm of flying insects are flying around a lamp. If there are 1000 insects, and each pair of insects has a probability of $1/1,000,000$ of bumping into each other every second, estimate the expected number of collisions after 1 minute?