Math 477, Final Exam Practice Sheet

1. Given 4 randomly chosen people,
   (a) what is the probability that they are all born on different days of the week?
   (b) Let X be the random variable representing the number of pairs of people born on the same day of the week. Find $E[X]$.

2. Suppose a random variable X represents the result of a die roll (possible values $1, 2, 3, 4, 5, 6$), and Y is the random variable which is 0 if the result of the roll is even and 1 if it is odd.

   Compute the covariance $Cov(X, Y)$.

3. Suppose that a person under 10 years of age has a 15% chance of having their shoelaces untied, and a person 10 years of age or older has a 3% chance of having their shoelaces untied. If 15% of the population is under the age of 10, what is the probability that a person is under the age of 10 given that their shoelaces are untied?

4. Suppose X is a random variable with probability density function $f(x) = 2 - 2x$ for $0 \leq x \leq 1$.
   (a) Compute the moment generating function for X.
   (b) Compute $Var(X)$.

5. Suppose X and Y are random variables with $Var(X) = 4$, $Var(Y) = 1$ and $Var(X - Y) = 3$. Is it possible that X and Y are independent? Why or why not?

6. Suppose that we are given a die which is possibly not fair, and that we don't know the likelihood of any of the results $\{1, 2, 3, 4, 5, 6\}$. Let X be the random variable representing the result of a roll. If we know that $E[X] = 3$, show that $P(X = 6) \leq 1/2$.

7. Let $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$ be the cumulative distribution function of the unit normal random variable.

   Suppose that a coin is flipped 1000 times, and let X be the number of heads. Use the central limit theorem to estimate the probability that $X < 550$. You may give your answer in terms of Φ.

8. Let X be a random variable with mean μ and variance σ^2. Show that $P(|X - \mu| > 2\sigma) \leq 1/4$.

9. Suppose that the temperature (in Fahreinheit) on a given day is a random variable given by a uniform distribution on $[0, 100]$. In a sequence of 7 days, what is the expected number of days which are colder then the day before them?

10. Suppose that the temperature (in Fahreinheit) on a given day is a random variable given by a uniform distribution on $[0, 100]$. What is the probability that the temperature on a given day is colder than both the day before and the day after.

11. Suppose that the temperature (in Fahreinheit) on a given day is a random variable given by a uniform distribution on $[0, 100]$. If the temperature on day 2 is colder than day 1, what is the probability that the temperature on day 2 is also colder than on day 3?

12. On a given day, the expected number of times that I forget something important is 3. Suppose that this is described by a Poisson random variable.
    (a) What is the probability that on a given day I forget 5 things?
    (b) What is the probability that on a given day I forget 5 things if we know that I have forgotten at least 4 things?

13. Suppose that on day 1, $3n$ people are divided up into n teams of 3 people each. On day 2, the people are randomly re-assigned into possibly different teams of 3, any grouping being equally likely.

    Let X be the number of people who have some teammate in common on both day 1 and day 2. What is the expected value of X?