## **Mathematical Statistics** Lecture 1: Probability review, part 1

Danny Krashen, Fall 2020

Basic objects & probability; Sample space S = (nesults of some experiment) Probability Passigns a neal number in [0,1] to certain subsets ECS P(E) Subsets ECS = event = callectron of potential

P(E) = probability at obtaining an astrane in E

Axions

 $P(\phi) = O$ P(S) = 1EinEj=Ø (motvelly exclusive) if Eics  $P(UE_i) = \sum_i P(E_i)$ Hen Usifil notron i me saz E, FCS am independent Hhat's it. it P(EvE)=D(E)D(E)



Random varables Det Arandon variable is a forcton X: S->IR ex: S = person chosen at random X=ht of a poson Notation: P(XEx) = P(ZSES/X(s) Ex]) smilarly P(x < X) P(a < X < b) etc.

Fonctions of a random ranable

Def: if gill -> ik ne mk g(X) for the new random varable gaX

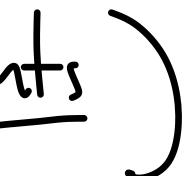




How to encade the information of a random virible? Comolatre Distubution function (c.d.f.): Fax) = P(X5x) (Discrete) Probability distribution: p(x) = P(X = x)(Controws) Probability density function (p. 24.) is an flu such that  $P(a \le X \le b) = \int_{a}^{b} f(x) dx$ 



Other way to encade: Moments) Farst: Expectator discrete core E[X]= Sxp(x) contrass care Laf Gadx Propert: E[X+Y]=E[X]+E[Y] E[YX] = YE[X]



Expectation of fews of andor ver.

Fad: E[g(x)] = Zg(x)f(x) Jisnek

= Sglafferdx Cantinueus

Moments Def the its moment of X about a is E[(X-a)]  $e_{X}$ :  $X \longrightarrow f_{W} = \begin{cases} 1 & o \le x \le 1 \\ 0 & e \le x \end{cases}$ othmoment conjudice =  $E[(X-a)^{\circ}] = E[1] = 1$ 1st monant about 0 =  $E[X] = \int_{0}^{1} x \cdot 1 = \frac{1}{2}$  $Z^{n}$  moment about  $1 = E[(X-I)^{2}] = \int (x-I)^{2} dx = ... = \frac{1}{3}$ 

Moments encade vez important i to Det M=mean=E[X] Det d<sup>2</sup> = variance = E[(X-n)<sup>2</sup>] Det  $M_i$  = ith moment about the mean  $(M_2 = \sigma^2)$ Def  $M_i^{\prime} = i^{th} moment about 0 (M_i^{\prime} = M = E[X])$ 



Facti (Usvelly), X is determined by its moments about 0

M., M., M., --- a list ef #s. Useful to "package" these into a permit serves  $M_{\chi}(t) = M_0^2 + M_1 t + M_2 \frac{t^2}{2} + M_3^2 \frac{t^3}{3!} + M_4^2 \frac{t^4}{4!} + \cdots$ "moment geverat functor

So  $M_i^2 = \frac{d^i}{dt_i} M_x(t) \Big|_{t=0}$ => X determed by  $M_x(t)$ . Alterrate descriptor if  $M_{\chi}(t) = E[e^{t\chi}]$  $M_{X+Y}(t) = M_{X}(t) M_{Y}(t)$ Nice joguit Vey useft toal for identify random sers.

