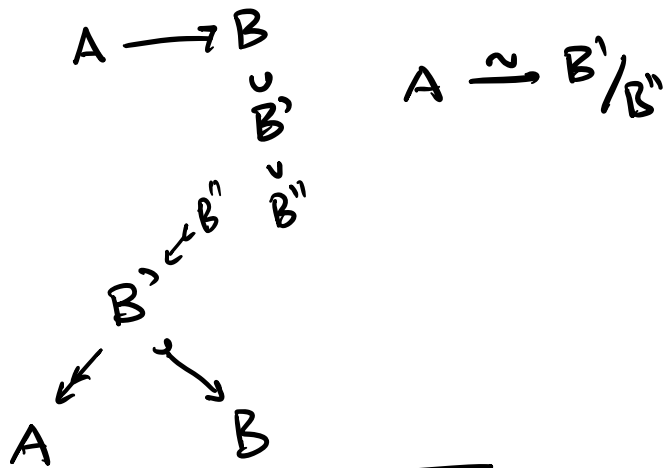


Recall:

Ingredients from last time

$\mathcal{C} \rightsquigarrow \mathcal{QC}$   
exact objects =  $oh(\mathcal{C})$   
morphisms  $\leftrightarrow$  subquotient resolutions



Nerve construction

$\mathcal{C} \rightsquigarrow NC$  simplicial complex

$NC_0 = oh(\mathcal{C})$

$NC_1 = \text{morphisms}$

$NC_i = a_0 \rightarrow a_1 \rightarrow \dots \rightarrow a_i$

classifying space

$BC \equiv |NC|$

Def  $K_i(C) = \pi_{i+1}(BQC, [0])$   
 $\uparrow$   
 exact  $0 \in ob C = ob(QC)$   
 "( $NQC$ )<sub>0</sub>"

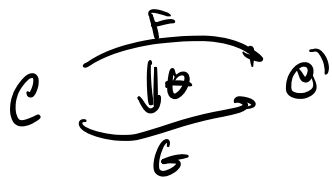
Rem.  $K_0(C) \xrightarrow{\sim} \pi_1(BQC, [0])$   
 $[A] \longmapsto [0] \xrightarrow{\quad} [A]$

Properties of  $B$  which we'll use later

$F: C \rightarrow C'$  functor

$\rightsquigarrow BF: BC \rightarrow BC'$

if  $\theta: F \Rightarrow G$  nat trans then  $B\theta$  is a homotopy between  $BF \neq BG$



$B\theta: BC \times I \rightarrow BC'$   
 homotopy between  $BF \neq BG$

$C \times \mathcal{C} \xrightarrow{\quad} \mathcal{C}$   
 functors are same as  
 pair of functors w/  
 nat trans between  
 them  
 $\mathcal{C} \text{ cat w/ } \circ \rightarrow \circ$

$B(D_1 \times D_2) = BD_1 \times BD_2$   
 $\uparrow$   
 product in cat of compactly gen spes.

(Compactly gen spes)  $\longleftrightarrow$  (Top spes)

$U \text{ open} \Leftrightarrow U \text{ ac}$   
 open in  
 all  $\mathcal{C}$   
 compact

If a functor has a (left or right) adjoint  $\Rightarrow$   
 hom. equivalence.

In particular, if  $\mathcal{C}$  has an initial or final object  
 $\Rightarrow BC$  is contractible.

Reminder: If  $X \xrightarrow{f} Y$  cont map, get  $\mathcal{C}$   
 define Homotopy fib  $F(f, y)$  to be the spec

$$\{(x, p) \in X \times \text{Paths in } Y \mid f(x) = p(y)\}$$

→ exact sequences in homology groups

$$\rightarrow \pi_{i+1}(Y, y) \rightarrow \pi_i(F(f, y), x) \rightarrow \pi_i(X, x) \rightarrow \pi_i(Y, y) \rightarrow$$

$y = f(x)$

---

Main result:  $C = \text{Coh-}\mathcal{O}_X\text{-mod}$   $X$  reg. scheme  
 $\text{Lac-free } \mathcal{O}_X\text{-mod}$

$$K_i(\text{Coh}_{\mathcal{O}_X}) = K_i(X)$$

$$K_i(\text{Lacfree}_{\mathcal{O}_X}) = G_i(X)$$

Given  $X, Y$  Noether scheme

$$f: X \rightarrow Y$$

$$f^*: LF(Y) \rightarrow LF(X)$$

is an exact fun.

$$\rightarrow \text{induced } \mathcal{O}LF(Y) \rightarrow \mathcal{O}LF(X)$$

$$\rightarrow f^*: K_i(Y) \rightarrow K_i(X)$$

on other hand, if  $f: X \rightarrow Y$  flat,

also get  $f^*: \text{Coh}(Y) \rightarrow \text{Coh}(X)$  exact

$$\Rightarrow f^*: G_i(Y) \rightarrow G_i(X)$$

alternately, if  $X$  is regular, <sup>Math</sup> get  $G_i(X) = K_i(X)$

since (by ex. in Hartshorne) any coh. sheaf

has a finite res by L. free sheaves

"Res. theorem"

$$K_i(LF(X)) = K_i(\text{Coh}(X))$$

## Push forwards

$f: X \rightarrow Y$  proper morphism

$\exists \text{ coh} \rightsquigarrow f_* \exists \text{ coh.}$   $R^i f_* \exists \text{ coh.}$

let  $\text{Coh}(X, f) =$  sub cat of coh sheaves  $\mathcal{F}$   
 $\Leftarrow R^i f_* \mathcal{F} = 0 \quad i > 0$

$\Rightarrow f_*: \text{Coh}(X, f) \rightarrow \text{Coh}(Y)$  is exact.

$$\Rightarrow f_*: K_i(\text{Coh}(X, f)) \rightarrow K_i(\text{Coh}(Y))$$

show (if  $X$  is q. projective) that any object  
in  $\text{Coh}(X)$  is resolvable by objects in  
 $\text{Coh}(X, A)$

$$\Rightarrow K_i(\text{Coh}(X, A)) \cong K_i(\text{Coh}(X))$$

$$\Rightarrow f_*: G_i(X) \rightarrow G_i(Y)$$