Topics in Applied Algebra, Exam 1 Practice Sheet

Helpful formulas

$$\text{Discrete Fourier Transform} \quad \hat{f}[k] = \sum_{j=0}^{N-1} \omega^{-jk} f[j]$$

$$\text{Fast Fourier Transform} \quad \hat{f}[k] = \widehat{f_{\text{even}}}[k] + \omega^{-k} \widehat{f_{\text{odd}}}[k]$$

$$\text{Basic Waveform Signals} \quad E_k = \sum_{j=0}^{N-1} \omega^{jk} e_j$$

1. Suppose we have a signal $f \in \ell_{\mathbb{C}}[\mathbb{Z}/4\mathbb{Z}]$. Let F_2 be the 2×2 Fourier matrix, and suppose that

$$F_2 \vec{f}_{\text{even}} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \; F_2 \vec{f}_{\text{odd}} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$

   Find $F_4 \vec{f}$.

2. Express the complex number $2e^{i\pi/6}$ in the form $a + bi$ where $a, b \in \mathbb{R}$.

3. Express $1 - i$ in the form $re^{i\theta}$ for real numbers r, θ.

4. Find a 3×3 matrix T such that for $\vec{f} = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$, we have $T\vec{y} = \overrightarrow{f * g}$.

5. Suppose $N = 2M$. Show that $e_M * E_1 = -E_1$.

6. Let S be the shift operator on $\ell_C[\mathbb{Z}/N\mathbb{Z}]$. Show that for a signal y, we have $S^2 y = e_2 * y$, where e_2 is the standard basis vector.

7. Let $f = 2e_0 - e_2$. Consider the linear operator T on $\ell_C[\mathbb{Z}/4\mathbb{Z}]$ defined by $Ty = f * y$. Exhibit a complete set of eigenvectors for T, and find thier eigenvalues.

8. Suppose that, when sampled at a rate of $N = 8$, a particular signal is given as: $f = 3e_0 - 2e_1 + e_4 - 7e_7$. What would the signal look like, in terms of the standard basis, if sampled at a frequency of $N = 4$?

9. Suppose that, when sampled at a rate of $N = 8$, a particular signal is given as: $f = 2E_0 - 2E_1 + E_4 - 7E_7$. What would the signal look like, in terms of the basis of waveforms, if sampled at a frequency of $N = 4$?