Wavelet transforms for unbounded signals part 2: towards wavelets from filters

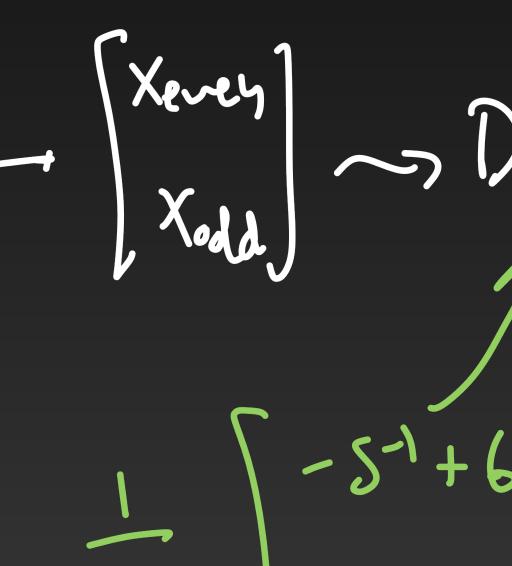


Applying our wavelet transforms to unbounded signals

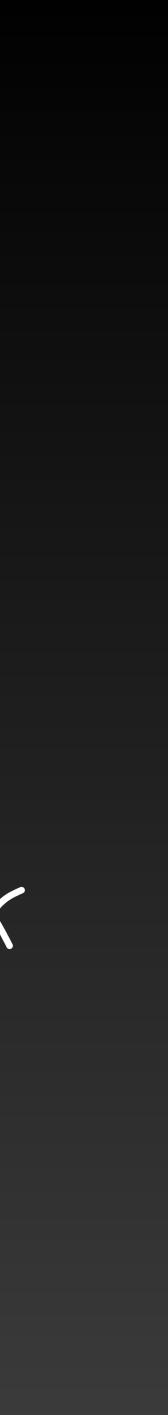
 $l(z/N) \sim l_o(z/)$

4P

S (21/N2L) case 1 Haar $X \rightarrow \begin{bmatrix} Xenen \\ Xoda \end{bmatrix} \sim \frac{1}{2} \begin{bmatrix} I & I \\ I & -I \end{bmatrix} \begin{bmatrix} Xenen \\ Xodd \end{bmatrix} = \begin{bmatrix} S \\ d \end{bmatrix}$ CDF(2,2) XI (Xever) ~> DUR [Xever] = [d] Xodd] ~> DUR [Xodd] = [d] $\frac{1}{1} \int -5^{-1} + 6\overline{P} - S \qquad 2\overline{P} + 2S \int \frac{1}{1} \int$ 452 - 28-1-2I



I(Z) car Xo=ZDX $\frac{\partial ef}{\partial ef} \times_{z_{v}} [k] = \chi [2k]$ "X even" and $[2i]: l_{0}(2i) - l_{0}(2i)$ $X, = ZJ S^{-1}X$ "X " ad $\frac{DeF}{SX}[k] = x[k-i]$



 $X_{0}[k] = (2\sqrt{X})[k] = X[2k]$ $X_{k} = (ZV SX) = (S'X) (Zk)$ = X = X [Zk+1]

Use these like Xeven ? Xodd.

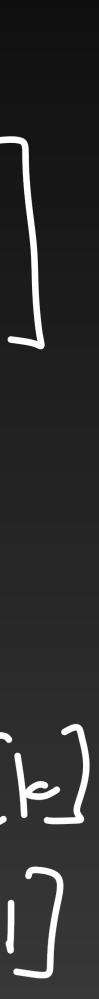
Standard Wavelet Transforms on Lo(Z) Haas: $X \sim 7 \begin{pmatrix} X_{o} \\ X_{i} \end{pmatrix} \sim \frac{1}{2} \begin{bmatrix} \Gamma & \Gamma \\ -\Gamma & \Gamma \end{bmatrix} \begin{pmatrix} X_{o} \\ X_{i} \end{bmatrix} = \begin{pmatrix} S \\ -L & \Gamma \end{bmatrix} \begin{pmatrix} X_{o} \\ X_{i} \end{bmatrix} = \begin{pmatrix} S \\ -L & \Gamma \end{bmatrix} \begin{pmatrix} X_{o} \\ X_{i} \end{bmatrix} = \begin{pmatrix} S \\ -L & \Gamma \end{bmatrix} \begin{pmatrix} X_{o} \\ X_{i} \end{bmatrix} = \begin{pmatrix} S \\ -L & \Gamma \end{bmatrix} \begin{pmatrix} X_{o} \\ X_{i} \end{bmatrix} = \begin{pmatrix} S \\ -L & \Gamma \end{bmatrix} \begin{pmatrix} X_{o} \\ X_{i} \end{bmatrix} = \begin{pmatrix} S \\ -L & \Gamma \end{bmatrix} \begin{pmatrix} X_{o} \\ X_{i} \end{bmatrix} = \begin{pmatrix} S \\ -L & \Gamma \end{pmatrix} \begin{pmatrix} X_{o} \\ X_{i} \end{bmatrix} = \begin{pmatrix} S \\ -L & \Gamma \end{pmatrix} \begin{pmatrix} X_{o} \\ X_{i} \end{bmatrix} = \begin{pmatrix} S \\ 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 $= \frac{1}{2} \left(X \left[2k \right] + X \left[2k \right] \right)$



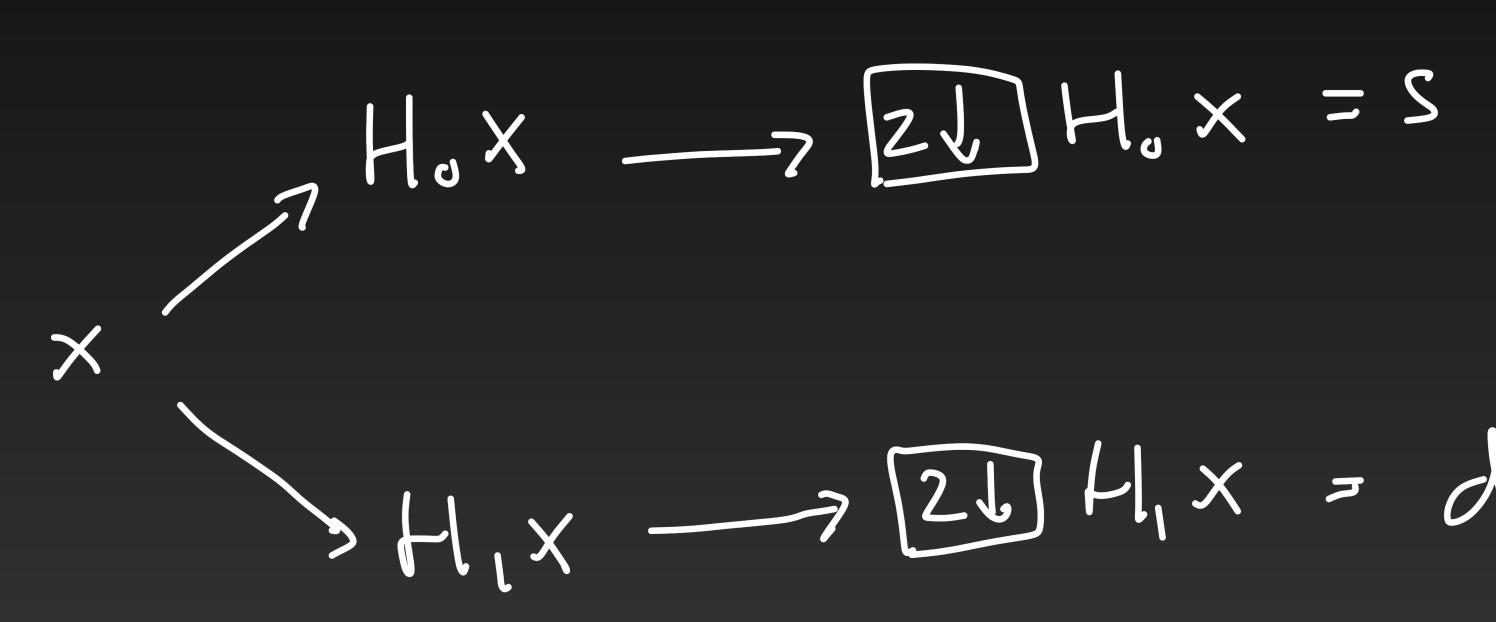
 $COP(2,2): X \sim 7 \begin{pmatrix} X_0 \\ X_1 \end{pmatrix} \sim 7 \frac{1}{4\sqrt{2}} \begin{bmatrix} -5^{-1}+6T-S & 2T+2S \\ -2S^{-1}-2T & 4T \end{bmatrix} \begin{bmatrix} x_0 \\ X_1 \end{bmatrix}$ 5 0 1 $\sim s[k] = -S'x_{s}[k] + 6x_{s}[k] - Sx_{s}[k] + 2x_{s}[k] + 2Sx_{s}[k]$ $= - \times [2k+2] + 6 \times [2k] - \times [2k-2] + 2 \times [2k+1] + 2 \times [2k-1]$

Standard Wavelet Transforms en lo(Z)



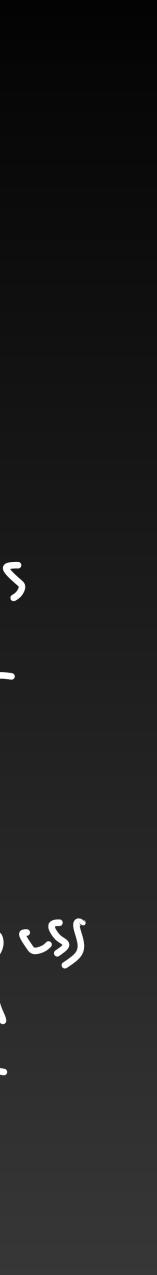
Bit why?

Alternate construction af waveletsi



 $H_0 = 10009555$ filts

H, = hrzhpess filt $>H_1 \times \longrightarrow 2J_1 H_1 \times = d$



Filturs (FIR = finite impulse response) Described by convolutions メーラル米ス $\frac{Def}{h*x} = \sum_{n} \left(\sum_{j+k=n} h[j]x[k] \right) e_{n}$ Alternately Def ej * e_k = ej + k ; foil (distribute)

 $= e_0 - e_2 + 3e_3 - 2e_5 - e_7$

 $(e_{-1} + 3e_{2} + e_{4}) * (e_{1} - e_{3})$

 $= \underbrace{e_{-1} * e_{1} - e_{-1} * e_{3} + 3e_{2} * e_{1} - 3e_{2} * e_{3} + e_{4} * e_{1}}_{e_{0}}$ $= e_0 - e_2 + 3e_3 - 3e_5 + e_5 - e_7$



exi

 $\Lambda_{o} = \mathcal{C}_{-1} + \mathcal{C}_{o}$ $h_1 = e_{-1} - e_0$

 $S = [Z](h_{o} * x)(k) = x[2k] + x[2k+1]$ d = [2](h, *x)[k] = -x[2k] + x[2k+1]

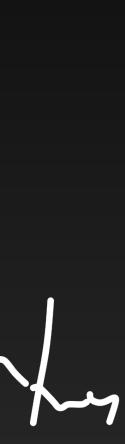
h,*×=-×+S×

h.*X = X + SX

 $\chi(z) + l(z) = z - transform if x + h$

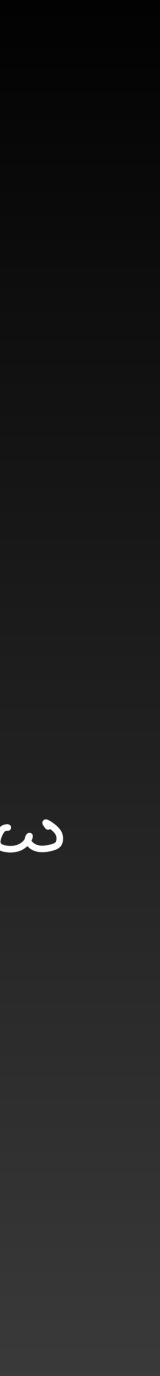
=> Frequencies are affected by h in proportion to their presence in h.

If we have a filt lo(21) -> lo(22), gren by h



 $h_{o} = e_{-1} + e_{o} \longrightarrow H_{o}(z) = z + 1$ 1997 ! $h_{1} = e_{1} - e_{0} \sim H_{1}(z) = z^{-1} - 1$ Power spectral densites: $P_0(\omega) = |H_0(e^{i\omega})|^2 = (e^{i\omega} + 1)(e^{i\omega} + 1) = 2 + 2\cos\omega$ w = 0 low $P(\omega) = 2 - 2\cos\omega$

w=IT high



Haar inconvenieure Ho cuts all highest freq. nell cuts all high-ish freq 50-50 H, --- similar. Questi Find filters which do this better ; which are neversible!