RESEARCH STATEMENT

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My research is in algebra and algebraic arithmetic geometry, particularly focusing on topics which relate to field arithmetic, noncommutative algebra and the Brauer group. My methods have primarily involved the use of tools and techniques from algebraic geometry, and ideas from analysis and topology in order to obtain results in algebra and field arithmetic.

My recent research has focused on the following topics:

1. *Finite dimensional algebraic structures:* central simple algebras, quadratic forms and algebraic with involution, and more generally, torsors for linear algebraic groups;
2. *Arithmetic of general fields:* field invariants, such as Galois cohomology groups, period-index problems, symbol length, generic splitting fields, Diophantine and cohomological dimensions;
3. *Arithmetic of semiglobal fields:* field patching, local-global principles for algebraic structures over such fields;
4. *Moduli stacks and derived categories of sheaves and twisted sheaves:* derived equivalences, index reduction for Brauer classes, Ulrich bundles, arithmetic of genus 1 curves, Weil-Chatêlet groups of Abelian varieties;

and these areas represent areas of active inquiry of my current research as well. Before discussing these topics in a little bit more detail below, let me briefly mention what other things I am thinking about, in my work in progress. My other current projects involve topics in noncommutative geometry (specifically, noncommutative birational geometry), algebraic classifying spaces, modular representation theory and support varieties, quadratic forms and algebras with involution, splitting fields of algebras in bad mixed characteristic, computing torsion in Chow groups of homogeneous varieties, Fulton-MacPherson configuration spaces, splitting fields of Albert and Octonion algebras, and new abstract contexts for local-global principles.

Finite dimensional algebraic structures were the original focus of my research. I began my studies by investigating the use of methods from algebraic geometry in the study of finite dimensional division algebras.

Division algebras, and more generally, central simple algebras, turn out to have critical importance in a number of areas: relating to the Tate conjecture and the Tate-Shafarevich groups, obstructions to local-global principles, problems of rationality in algebraic geometry, gerbes and hence obstruction classes for various naturally arising moduli stacks.

My research on central simple algebras and related structures has used, as its main method, the development and enrichment of dictionaries between these structures and the algebraic geometry of homogeneous varieties. In this way, more novel approaches in arithmetic and algebraic geometry may be brought to bear on the problems in algebra.

My contributions in this area include:

- new cases of and generalizations of a conjecture on Amitsur from the 1950's, concerning the birational equivalence of Severi-Brauer varieties [Kra03, Kra08];
- results on common splitting fields of collections of central simple algebras, and splitting fields for separable algebras, generalizing classical results of Albert and Risman [Kra10a];
- results on when one can distinguish division algebras by their finite splitting fields (joint with Kelly McKinnie) [KM15];
- determination of the Chow group of zero cycles on certain homogeneous varieties [Kra10c];
- descriptions of the Chow Motives of unitary and orthogonal homogeneous varieties [Kra07];
- work on birational geometry of symmetric powers of varieties, particularly Severi-Brauer varieties (joint with David J. Saltman) [KS04].

The arithmetic of general fields has seen vast advances in recent years, with the advent of motivic homotopy theory and the solutions of the Milnor and Bloch-Kato conjectures (see [MV99, Voe03a, Voe03b, Voe11, Wei09, OVV07]). These powerful theorems have led to more refined questions about Galois Cohomology (such as the symbol length problem), and have also lent us new methods by which we may attempt to answer them.

The problem of determining symbol length and indices for cohomology classes, and of constructing generic splitting varieties, is a topic of central importance in field arithmetic and ties in with other work in essential dimension of algebraic stacks (see for example, [BRV10, Kah00, KM03, CM13, CM14]).

My work in this area has included:

- clarifying the relationship between different measures of dimension of fields and complexity of cohomology classes, and in particular, showing that the index of a cohomology class may be bounded in terms of the Diophantine dimension [Kra13];
- giving explicit linear bounds on the p-cohomological dimension of a field in terms of its Diophantine dimension, answering a question of Serre in certain cases (joint with Eli Matzri) [KM15].

A semiglobal field is the function field of a curve over a complete discretely valued ground field. **The arithmetic of semiglobal fields** has been studied in a number of contexts; in order to understand higher class field theory [Kat86], in its relationship to Tate-Shafarevich groups [Gro68, PS96], and as an important ingredient in understanding torsors over finitely generated fields via local-global principles [PP03, LPS14]. As semiglobal fields are naturally the function fields of two-dimensional schemes, algebraic structures over semiglobal fields have been of great interest as an important step to understanding the behavior of such objects in more general higher-dimensional contexts.

Field patching is a method for studying the arithmetic of semiglobal fields, introduced initially by Harbater and Hartmann in 2006, and developed subsequently by myself, Harbater and Hartmann jointly from 2007. These methods have been featured in a number of past and upcoming workshops and conferences. For example, a summer school program in Heidelberg in 2008, an AIM workshop in 2011, the 2012 Arizona Winter School, a 2012 workshop in Essen, Germany, a workshop in 2015 at the University of Pennsylvania ("Local-global principles and their obstructions"), and an upcoming workshop in 2016 at Emory University ("Emory conference on higher obstructions and rational points").

My work on semiglobal fields and field patching includes:

- obtaining bounds for u-invariants, and solutions to the period index problem for various semiglobal fields (joint with David Harbater and Julia Hartmann) [HHK09];
- the admissibility problem for semiglobal fields – which groups can be realized as Galois groups of maximal subfields of division algebras, (originally posed by M. Schacher for global fields in [Sch68], see also [FS95, Fei02, NP10, RS13]), (joint with David Harbater and Julia Hartmann) [HHK11];
- development of local-global principles for torsors for linear algebraic groups and Galois cohomology classes over semiglobal fields (joint with David Harbater and Julia Hartmann) [HHK15, HHK14];
- extending local-global principles from rational to retract rational linear algebraic groups over semiglobal fields [Kra10b].

Moduli stacks and derived categories of sheaves and twisted sheaves have enjoyed a rich connection to the theory of the Brauer group. The somewhat modern perspective that Brauer classes arise naturally as obstruction classes for the fine-ness of various moduli spaces of sheaves can be seen implicitly in many places, from Tate's pairing [Tat58], to Grothendieck's study of genus 1 fibrations [Gro68], and in explicit form in more modern references such as [HS05, Kha05, BBGN07, Lie13]. These natural obstruction classes have, in certain cases, also been shown to be realizable by generalized Clifford algebras [Hai84, Hai92, Kul03, CKM12], providing particularly concrete descriptions in these situations.

My work in this area includes:

- giving a generalized notion of a Clifford algebra associated to finite morphism, with applications to the period-index problem for genus 1 curves (joint with Max Lieblich) (preprint);
- determining when the derived categories for torsors for Abelian varieties are equivalent, via DG-categorical methods (joint with Benjamin Antieau and Matthew Ward), (under revision in Advances in Mathematics);
- providing new interpretations of the Tate pairing, and algorithms for computing Relative Brauer groups of genus 1 curves, (joint with Mirela Ciperiani) [CK07];
- computing index reduction formulas for Brauer classes with respect to function fields of genus 1 curves, using moduli spaces of twisted sheaves and their obstruction classes (joint with Max Lieblich, with an appendix by Bhargav Bhatt) [KL08].

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