Math 2260, Fall 2014, Practice Sheet for Exam 3

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1. Compute the limits of the following sequences as n approaches ∞. Justify your answers!
   a. $a_n = \dfrac{2^n}{n!}$

   b. $a_n = \dfrac{2^n}{2^{n+1}}$

   c. $a_n = \dfrac{2^n}{n^5}$

   d. $a_n = \left(1 - \dfrac{1}{n}\right)^{2n}$

2. Use any method of your choice to determine which of the following series converges or diverges:
   a.

   b. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$

   c. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

   d. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + 4n + 1}$

   e. $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n}$

   f. $\displaystyle\sum_{n=1}^{\infty} 3^n$

   g. $\displaystyle\sum_{n=1}^{\infty} ne^{-3n}$

h. $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{2n}}$

i. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n}$

j. $\displaystyle\sum_{n=1}^{\infty} e^{-3n}$

3. For at least two of the series in the previous problem, you should be able to compute the actual number to which the series converges. Find two series above which converge, and compute the values to which they converge.

4. For which values of x do the following power series converge?
   (a)
   $$\sum_{n=0}^{\infty} n3^n x^{2n}$$
   (b)
   $$\sum_{n=0}^{\infty} \frac{1}{n} \left(\frac{1}{2}\right)^n x^n$$
   (c)
   $$\sum_{n=0}^{\infty} \frac{1}{n^2} 5^n (4x-3)^n$$

5. Write the first 7 terms for the Taylor series expansion for the function $f(x) = e^{2x}$ about $x = 0$.

6. Write the first 3 terms for the Taylor series expansion for the function $f(x) = e^{2x}$ about $x = 2$.