Math 2260, Spring 2014, Practice Sheet for Exam 1

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1. Find the area of the regions described below:
   a. The area between the graph of the function $2\sin x+3$ and the x-axis, for $0 \le x \le \pi/2$.
   b. The area between the graph of the function $\sin x$ and the x axis, for $0 \le x \le 2\pi$.
   c. The area between the graph of the function $\frac{1}{4-2x}$ and the x-axis, for $-1 \le x \le 1$.
   d. The area in the first quadrant bounded by the graphs of the functions $f(x) = x^2$ and $g(x) = x$.

2. Calculate the following indefinite integrals:
   a. $\displaystyle\int (\sin x)e^{\cos x}dx$
   b. $\displaystyle\int \frac{1}{x\ln x}dx$
   c. $\displaystyle\int \ln x^{1/x}dx$
   d. $\displaystyle\int \sin x\cos(\cos x)dx$
   e. $\displaystyle\int \sin^3 x\cos^3 x dx$
   f. $\displaystyle\int \frac{\sin(\ln x)}{x}dx$

3. Find the volume of the solid of revolution obtained by revolving the region in the first quadrant bounded by the lines $x = 2$ and $y = 2x$
   a. about the x-axis
   b. about the y-axis
   c. about the line $x = 3$
   d. about the line $y = 10$

4. Find the volume of a solid of revolution with the following decription:
   - the solid lies between the planes at $y = 1$ and $y = 3$,
   - for a given y-value, the cross-section is a square whose diagonal has length $\frac{\ln y}{y}$.

5. Find the volume of a solid of revolution with the following decription:
   - the solid lies between the planes at $y = 5$ and $y = 6$,
   - for a given y-value, the cross-section is an elliptical oval whose surface area is $y^2 + y$.

6. Write down an integral which would be used to compute the following volume of the solid of revolution:
   a. The region in the first quadrant bounded by the graphs $f(x) = x^2$ and $g(x) = x$ rotated about the y axis.
   b. The region in the first quadrant bounded by the graph $x = y - y^3$, rotated about the y-axis.

7. Write down an integral which expresses the arclength of the curve described by
   a. $x(t) = t \sin t, y(t) = t^2 \cos t, 0 \leq t \leq 2\pi$
   b. $x(t) = t^3 - t^2, y(t) = \sqrt{t}$
   c. $y = \sin x, 0 \leq x \leq \pi$

8. Write down an integral which expresses the surface area of the surface of revolution described by
   a. rotation of the curve $y = x \sin x, 0 \leq x \leq 2\pi$ about the x-axis
   b. rotation of the parametric curve $x = y^2 - \cos y, 0 \leq y \leq 2\pi$ about the y-axis

9. Write down the surface are of the surface of revolution obtained by rotating the line segment $y = 3x + 4, 0 \leq x \leq 1$ about the x-axis.

10. (*challenge!*) Calculate the volume of the solid described as follows:
    - the solid lies between the planes at $y = 1$ and $y = 4$,
    - for a given value of y, the cross-section at y is a rectangular region with side lengths $a(y)$ and $b(y)$,
    - the function $b(y)$ is the derivative of the function $a(y)$: that is, $\frac{d}{dy}a(y) = b(y)$,
    - $a(1) = 2, a(4) = 10$.

11. (*challenge!*) Use the formula for the surface area of a surface of revolution to derive the formula for the surface area of a sphere of radius r.