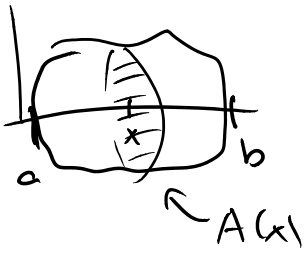
Lecture 7: shells

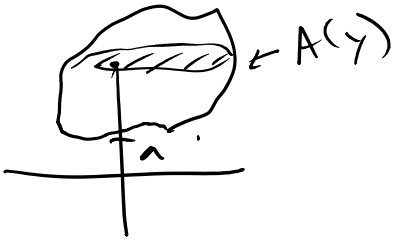
Tuesday, September 2, 2014 12:32 PM

Volumes of solids

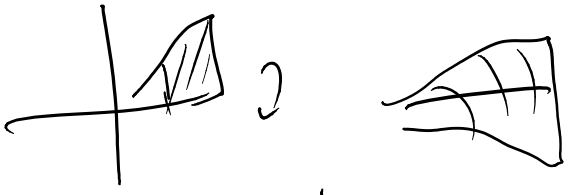
cross-section: $\int_{x=a}^{x=b} \text{Area}(x)\,dx$



$\int_{y=a}^{y=b} \text{Area}(y)\,dy$

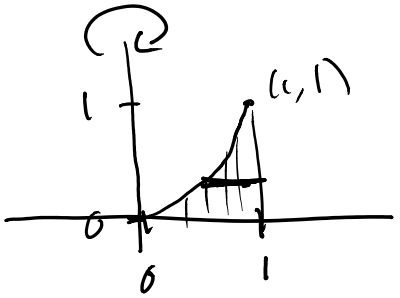


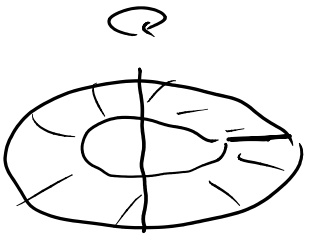
Revolution:



x-axis $\int_{x=a}^{x=b} A(x)\,dx = \int_{x=a}^{x=b} \pi r^2\,dx$

Area under $y = x^2$ revolving about y. between $x = 0, 1$





$\int_{y=1}^{y=1} \text{Area(washer at ht } y)\,dy = \int_{1}^{1} (\pi - \pi y)\,dy$

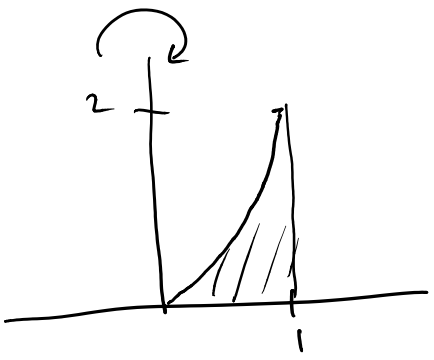
$$\int_{y=0}^{y=1} \text{Area(washer at ht } y)\, dy = \int_0^1 (\pi - \pi y)\, dy$$

$\pi(\text{outer rad})^2 - \pi(\text{inner rad})^2$

$\pi(1)^2 - \pi(\sqrt{y})^2$

$\sqrt{y}$ ← x-value

Ex: $y = x^2 + x$ under between $x = 0$ & 1 rotated about y



$$\int_{y=0}^{y=2} A(y)\, dy$$

$A(y) = \pi(\text{out rad})^2 - \pi(\text{in rad})^2$

$= \pi(1)^2 - \pi\left(\frac{-1+\sqrt{1+4y}}{2}\right)^2$

$$\int_0^2 \pi - \pi\left(\frac{\sqrt{1+4y}-1}{2}\right)^2 dy$$

$y = x^2 + x$

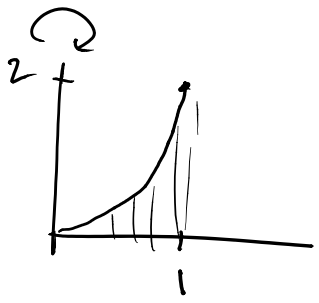
$x^2 + x - y = 0$

$x = \frac{-1 \pm \sqrt{1 - 4(1)(-y)}}{2}$

$= \frac{-1 \pm \sqrt{1+4y}}{2}$

$$\int_0^2 \pi\, dy - \pi \int_0^2 \left(\frac{\sqrt{1+4y}-1}{2}\right)^2 dy$$

$$x = \frac{-1+\sqrt{1+4y}}{2}$$



$y = x^3 + x$ between $x=0$ & $x=1$

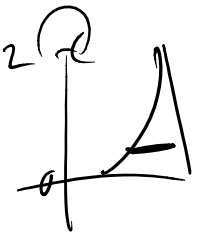
about y-axis

$$\int_{y=0}^{y=2} \pi 1^2 - \pi x^2\, dy$$

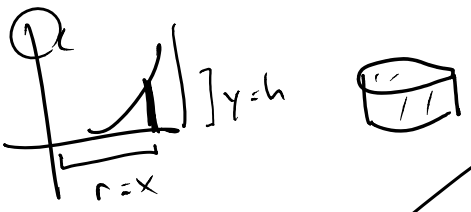
$x = ?$

$x^3 + x = y$ solve for x:

Cylinders



$$\int_{y=0}^{y=2} (\pi - \pi x^2)\, dy$$



$$\int_{x=0}^{x=1} 2\pi r h\, dx$$

$r = x$

$$\int_0^1 2\pi x\, dx$$

$r = x$
$h = y$

$$\int_0^1 2\pi x y \, dx$$

$$\int_0^1 2\pi x (x^3 + x) \, dx$$

$$2\pi \int_0^1 x^4 + x^2 \, dx = 2\pi \left(\frac{1}{5} x^5 + \frac{1}{3} x^3 \right)_0^1$$

$$= 2\pi \left(\frac{1}{5} + \frac{1}{3} \right)$$

$y = x^2 + x \quad 0 \le x \le 1$ about y-axis

$$\int_0^1 2\pi r h \, dx = \int_0^1 2\pi x (x^2 + x) \, dx$$

$$= 2\pi \int_0^1 x^3 + x^2 \, dx = 2\pi \left[\frac{1}{4} x^4 + \frac{1}{3} x^3 \right]_0^1$$

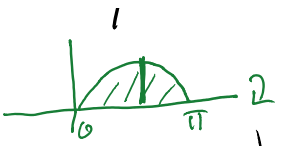
$$= 2\pi \left(\frac{1}{4} + \frac{1}{3} \right)$$

Practice: Setup integral – don't solve.

1. Area under $\sin x = y$ between $x = 0$ & $x = \pi$ about x-axis

Disks! $\int_0^\pi \pi \sin^2 x \, dx$

1. Area ...
about x-axis

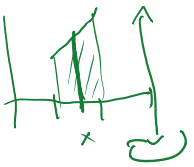


Disks! $\int_0^{\pi} \pi \sin^2 x \, dx$

2. Area under $\sin x = y$ between $x = 0$ & $x = \pi$
about y-axis

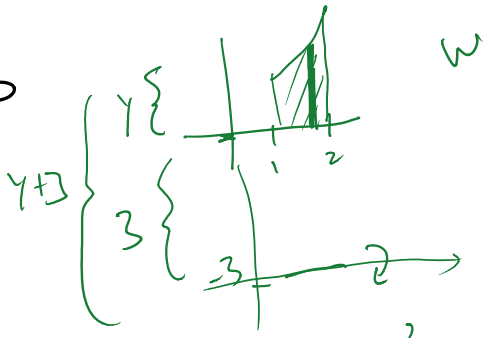
Shells! $\int_0^{\pi} 2\pi x \sin x \, dx$

3. $y = e^x$ between $x = 1$ & $x = 2$ about
$x = 3$



Shells: $\int_1^2 2\pi r h \, dx$
$3 - x = r$
e^x

4. $y = e^x$ between $x = 1$ & $x = 2$ about
$y = -3$



Washers:
inn radius = 3
out rad = $y + 3$

$\int_1^2 \pi (e^x + 3)^2 - \pi (3)^2 \, dx$