Lecture 6: more solids of revolution and stuff

Thursday, August 28, 2014 12:32 PM

Some webwork stuff

$$\int x^6 e^{x^7} dx = \frac{1}{7}\int e^u du = \frac{1}{7} e^u + C$$

$u = x^7$

$= \frac{1}{7} e^{x^7} + C$

$du = 7x^6 dx$

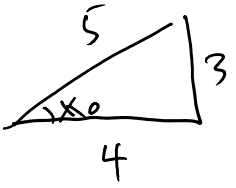
$\frac{1}{7} du = x^6 dx$

$y = 4\sin x \quad y = 3\cos x$ x between 0 & $.3\pi$ = 54°

1. when do graphs cross

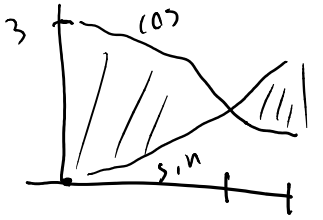
$4\sin x = 3\cos x$

$\tan x = \frac{3}{4}$



$x < 45°$

x_0 = the angle above



where $\tan x = \frac{3}{4}$

$3\pi/10$

where $\tan x = \frac{3}{4}$

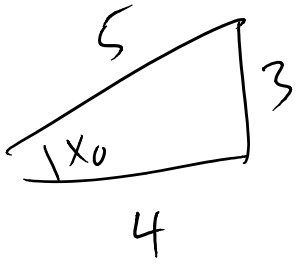
$$\int_0^{x_0} (3\cos x - 4\sin x)\,dx + \int_{x_0}^{3\pi/10} (4\sin x - 3\cos x)\,dx$$

$$\Big[3\sin x + 4\cos x\Big]_0^{x_0} + \Big[-4\cos x - 3\sin x\Big]_{x_0}^{3\pi/10}$$

$$(3\sin x_0 + 4\cos x_0 - 3\sin 0 - 4\cos 0) - 4\cos 3\pi/10 - 3\sin 3\pi/10$$

$$+ 4\cos x_0 + 3\sin x_0$$

$$6\sin x_0 + 8\cos x_0 - 4 - 4\cos 3\pi/10 - 3\sin 3\pi/10$$



$\sin x_0 = \frac{3}{5}$ $\cos x_0 = \frac{4}{5}$

$$6\cdot\frac{3}{5} + 8\cdot\frac{4}{5} - 4 - 4\cos 3\pi/10 - 3\sin 3\pi/10$$

$$\frac{18}{5} + \frac{32}{5} - 4 - 4\cos 3\pi/10 - 3\sin 3\pi/10$$

$$1 - 4\cos 3\pi/10 - 3\sin 3\pi/10$$

$6 - 4\cos^{3\pi/10} - 3\sin^{3\pi/10}$

$$\int_{\pi/2}^{\pi/6} \frac{\cos z}{\sin^6 z}\, dz$$

$u = \sin z$

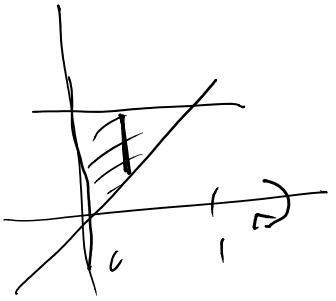
$du = \cos z\, dz$

$\frac{1}{\square^6}$ ⟹ $e^{\square}$

$$\int x^6 e^{x^7}\, dx$$

Revolve about x-axis:

$y = x \quad y = 1 \quad x = 0$



$$\int_0^1 (\pi 1^2 - \pi x^2)\, dx$$

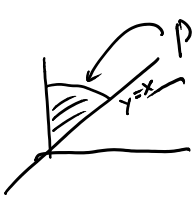
Practice 1. about x-axis

$y = x^2 \;/\; y = 0 \quad x = 2$

2. between $\sqrt{\cot x}$ & x-axis

$x = \pi/6$ & $x = \pi/2$ about x

3. $x = \tan\frac{\pi}{4}y$ between $y=0$ & $y=1$ about y-axis

4. 

part of circle of radius 3 about y-axis

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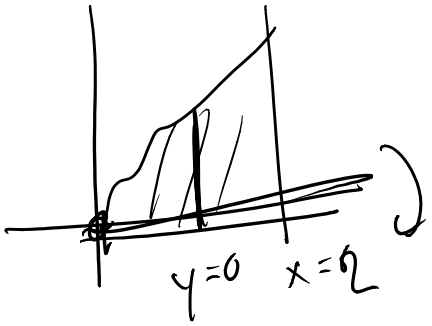
1. $y = x^2$ $y=0$ $x=2$

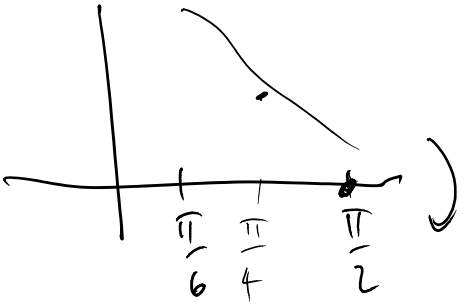


$y = x^2 = 0$

$x = 0$

$$\int_0^2 \pi (x^2)^2 \, dx$$

2. $y = \sqrt{\cot x}$ between x-axis & $x = \frac{\pi}{6}$ & $\frac{\pi}{2}$

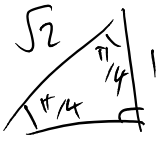


crosses?

$\sqrt{\cot x} = 0$?

$\cot x = 0$

$\frac{\cos x}{\sin x} = 0$



$\Rightarrow \cos x = 0$ $\frac{\pi}{2}$ only in

$\Rightarrow \cos x = 0$ $\frac{\pi}{2}$ only in our region

$$\int_{\pi/6}^{\pi/2} \pi r^2 \, dx = \int_{\pi/6}^{\pi/2} \pi \left(\sqrt{\cot x}\right)^2 dx$$

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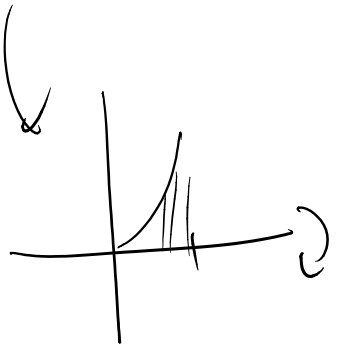
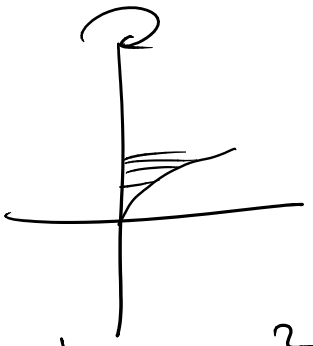
3. $x = \tan \frac{\pi}{4} y$ y between 0 & 1 about y-axis

$y = \tan \frac{\pi}{4} x$ x between 0 & 1 about x-axis



$$\int_0^1 \pi \left(\tan \frac{\pi}{4} x\right)^2 dx$$



$$\int_0^1 \pi \left(\tan \frac{\pi}{4} y\right)^2 dy$$

$$\int \tan^2 x \, dx = \int \frac{\sin^2 x}{\cos^2 x} \, dx \quad \text{... oh-oh}$$

$u = \tan x$

$du = \sec^2 x \, dx$

$u = \cos x$

$du = -\sin x \, dx$

$du = \sec^2 x\, dx$

stuck.

$\sin^2 x + \cos^2 x = 1$ $\quad / \cos^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$\int (\sec^2 x - 1)\, dx$

$= \int \sec^2 x\, dx - \int dx$

$(\tan x - x) + C$.