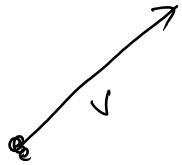
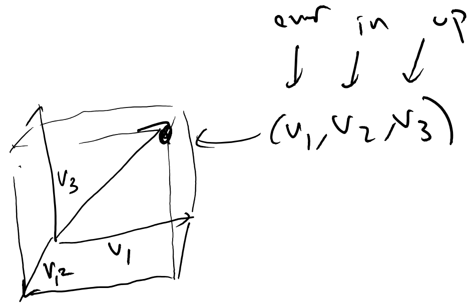
# Lecture 36: multiplying vectors

Thursday, November 20, 2014 12:46 PM

## Notations for vectors




other notation: $\vec{i}, \vec{j}, \vec{k}$ notation.

$\vec{i} = (1,0,0) \qquad \vec{j} = (0,1,0) \qquad \vec{k} = (0,0,1)$

$$\begin{aligned}(v_1, v_2, v_3) &= (v_1, 0, 0) + (0, v_2, 0) + (0, 0, v_3) \\ &= v_1(1,0,0) + v_2(0,1,0) + v_3(0,0,1) \\ &= v_1\vec{i} + v_2\vec{j} + v_3\vec{k}\end{aligned}$$

$$2\vec{i} + 3\vec{j} - 2\vec{k} = (2, 3, -2)$$

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## Multiplication of vectors

Scalar multiplication

$c \cdot \vec{v}$ = stretch $\vec{v}$ by a factor $c$

$c = \#$ $\quad \vec{v}$ vector

ex: $\quad c = 3 \quad \vec{v} = (7, 20, -2)$

$$c \cdot \vec{v} = (3\cdot 7,\ 3\cdot 30,\ 3\cdot(-2))$$

$$3\,(7\vec{i} + 20\vec{j} - 2\vec{k}) = 3\cdot 7\,\vec{i} + 3\cdot 20\,\vec{j} - 3\cdot 2\,\vec{k}$$

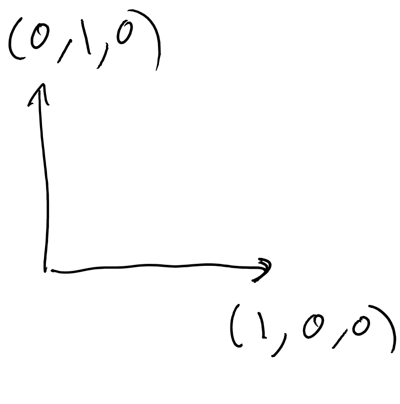
## Dot Product

$\vec{v}\cdot\vec{w}$ is a # which measures to what extent $\vec{v}$ & $\vec{w}$ point "same way"

$\vec{v} = (v_1, v_2, v_3) \qquad \vec{w} = (w_1, w_2, w_3)$

$$\vec{v}\cdot\vec{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$$

ex

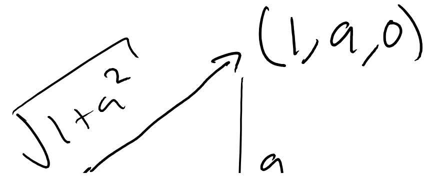



$(0,1,0)\cdot(1,0,0)$
$= 0\cdot 1 + 1\cdot 0 + 0\cdot 0$
$= 0$

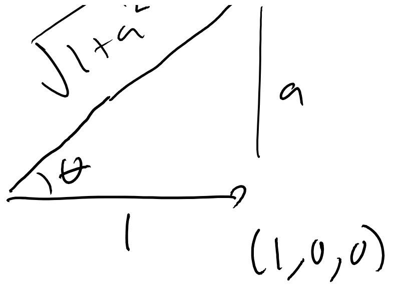
Fact: $\quad \vec{v}\cdot\vec{w} = |v|\,|w| \cos\theta$

$\theta$ angle between them.

ex:



ex:




$$(1,a,0)\cdot(1,0,0)$$
$$= 1+0 = 1$$

$$\cos\theta = \frac{1}{\sqrt{1+a^2}}$$

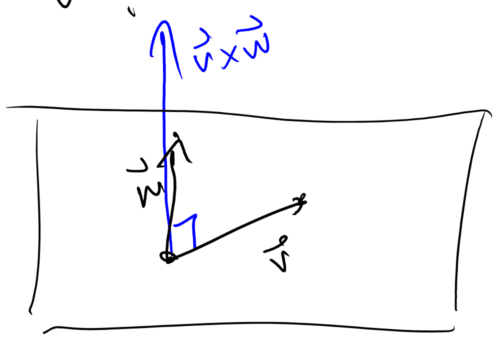
$$(1,a,0)\cdot(1,0,0) = \underbrace{|(1,a,0)|}_{\sqrt{1+a^2}}\ \cdot\ \underbrace{|(1,0,0)|}_{1}\ \cdot\ \underbrace{\cos\theta}_{\frac{1}{\sqrt{1+a^2}}}$$
$$\quad = 1$$

in particular, get: $\theta = \cos^{-1}\left(\dfrac{\vec{v}\cdot\vec{w}}{|\vec{v}|\,|\vec{w}|}\right)$

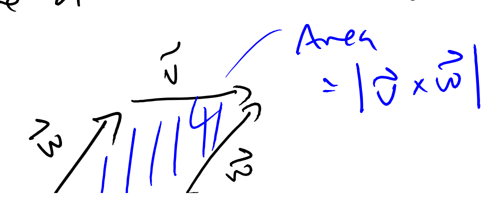
## Cross product:

$\vec{v}\times\vec{w}$ measures how perpendicular $\vec{v}$ & $\vec{w}$ are

is a vector, perpendicular to the plane that $\vec{v}$ & $\vec{w}$ lie in

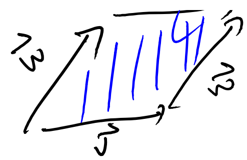



length of the cross product measures the area in the //-ogram




direction of $\vec{v}\times\vec{w}$
use the "... "

direction of …
use the
"right hand rule"




## Cross Product:

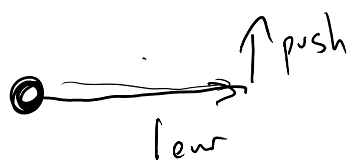
$$\vec{v} \times \vec{w} = (v_2 w_3 - v_3 w_2,\ v_3 w_1 - v_1 w_3,\ v_1 w_2 - v_2 w_1)$$

$$\vec{v} = (v_1, v_2, v_3)$$

$$\vec{w} = (w_1, w_2, w_3)$$

$$\vec{v} \times \vec{w} = \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{pmatrix} = \vec{i}(v_2 w_3 - v_3 w_2) - \vec{j}(v_1 w_3 - v_3 w_1) + \vec{k}(v_1 w_2 - v_2 w_1)$$

## Torque:

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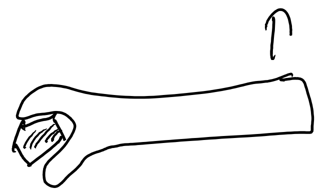

$$\text{torque} = \vec{d} \times F$$