Lecture 34: Using Taylor series. Function in more variables

Tuesday, November 18, 2014 12:28 PM

$\sum a_n x^n$ which x does this conv?

ratio test

$\sum \frac{1}{n} 2^n x^n$

$\lim_{n\to\infty} \frac{|a_{n+1}x^{n+1}|}{|a_n x^n|}$ → 1^∞ → logs

→ L'hopital

→ Squeeze

endpts $\int$ test?

→ partial fractions --

trig ↙ ↘ parts

Taylor Series to approximate anything.

$\sqrt{5} = ?$ $f(x) = \sqrt{x}$ know $f(4) = 2$

want $f(5)$

$f(x) \approx P_0 = f(4)$

$P_1 = f(4) + \frac{1}{1!} f'(4)(x-4)$

expansion about $x = 4$ "a"

$P_2 = f(4) + \frac{1}{1!} f'(4)(x-4) + \frac{1}{2!} f''(4)(x-4)^2$

$P_2(x) = 2 + \frac{1}{4}(x-4) + \frac{1}{2}\left(-\frac{1}{32}\right)(x-4)^2$

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$f(4) = 2$

$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$f''(4) = -\frac{1}{4}\frac{1}{(\sqrt{4})^3} = -\frac{1}{32}$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$f''(x) = -\frac{1}{4}x^{-3/2} = -\frac{1}{4(\sqrt{x})^3}$

.02 ish

$$\sqrt{5} = f(5) \approx P_2(5) = 2 + \frac{1}{4}(1) - \frac{1}{64}1^2$$

$$= 2.25 - .02 = 2.23 \text{ ish.}$$

Thm 2.4

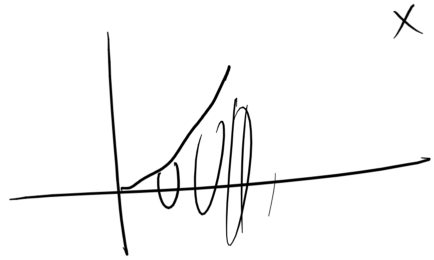
$$\sqrt{3} = f(3) \approx 2 + \frac{1}{4}(-1) - \frac{1}{64}(-1)^2 = 1.75 - \frac{1}{64}$$

1.73 ish

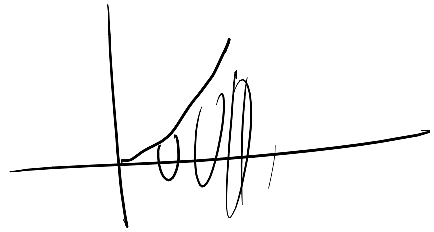
$$= \int_0^1 2\pi x^2 \sqrt{1+4x^2}\, dx$$

SA of revolution of $y = x^2$ about x axis $x = 0$ to 1

$$S(t) = \int_0^t 2\pi x^2 \sqrt{1+4x^2}\, dx$$



$$S(t) = \int_0^t 2\pi x^2 \sqrt{1+4x^2}\, dx$$



$S(1) = ?$

$$S'(t) = \underbrace{2\pi t^2}\,\underbrace{\sqrt{1+4t^2}}$$

$S(0) = 0$

$$P_2(t) = S(0) + S'(0)\,t + \frac{S''(0)}{2} t^2 \quad = 0$$

about $t = 0$

"a"

$S(0) = 0$

$S'(0) = 0$

$$S''(t) = 4\pi t \sqrt{1+4t^2} + \underbrace{2\pi t^2 \, \tfrac{1}{2}(1+4t^2)^{-1/2} \cdot 8t}_{8\pi t^3 (1+4t^2)^{-1/2}}$$

$S''(0) = 0$

try P_3

$$S'''(t) = 4\pi \sqrt{1+4t^2} + 4\pi t \left(\sqrt{1+4t^2}\right)' + 24\pi t^2 (\quad)^{*} + 8\pi t^3 (\quad)'$$

$$S'''(0) = 4\pi \sqrt{1} = 4\pi$$

$$P_3 = \frac{1}{3!} 4\pi t^3$$

$$P_3(1) \approx \frac{4}{6}\pi = \frac{2}{3}\pi$$

$T(x,y,z)$ temperature at a point (x,y,z)

ask: how does temp change as we move?

$$T(x,y,z) = x + 3y - 2z + y^2 - xz + 2$$

at $x=1, y=0, z=0$ $(1,0,0)$

$T(1,0,0) = 3$

how does it chg in x-direction?

consider the path: $(1+t, 0, 0)$

$$\frac{d}{dt} T(1+t,0,0) = \frac{d}{dt}(1+t+2) = 1$$

in y? in z?

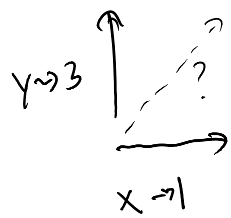
in y direction: $\frac{d}{dt}(T(1,t,0))$

$$\frac{d}{dt}(1+3t+t^2+2) = 3+2t$$

... what happens at the point $(1,0,0)$

$t = 0$

set $\boxed{3}$



$$\frac{d}{dt}T(1+t, t, 0) = 4!$$

Punchline: "The derivative" of T is a vector

∇T

change in direction v ⇝ given by $\nabla T \cdot v$