Lecture 33: Exam 3 Review, part 2

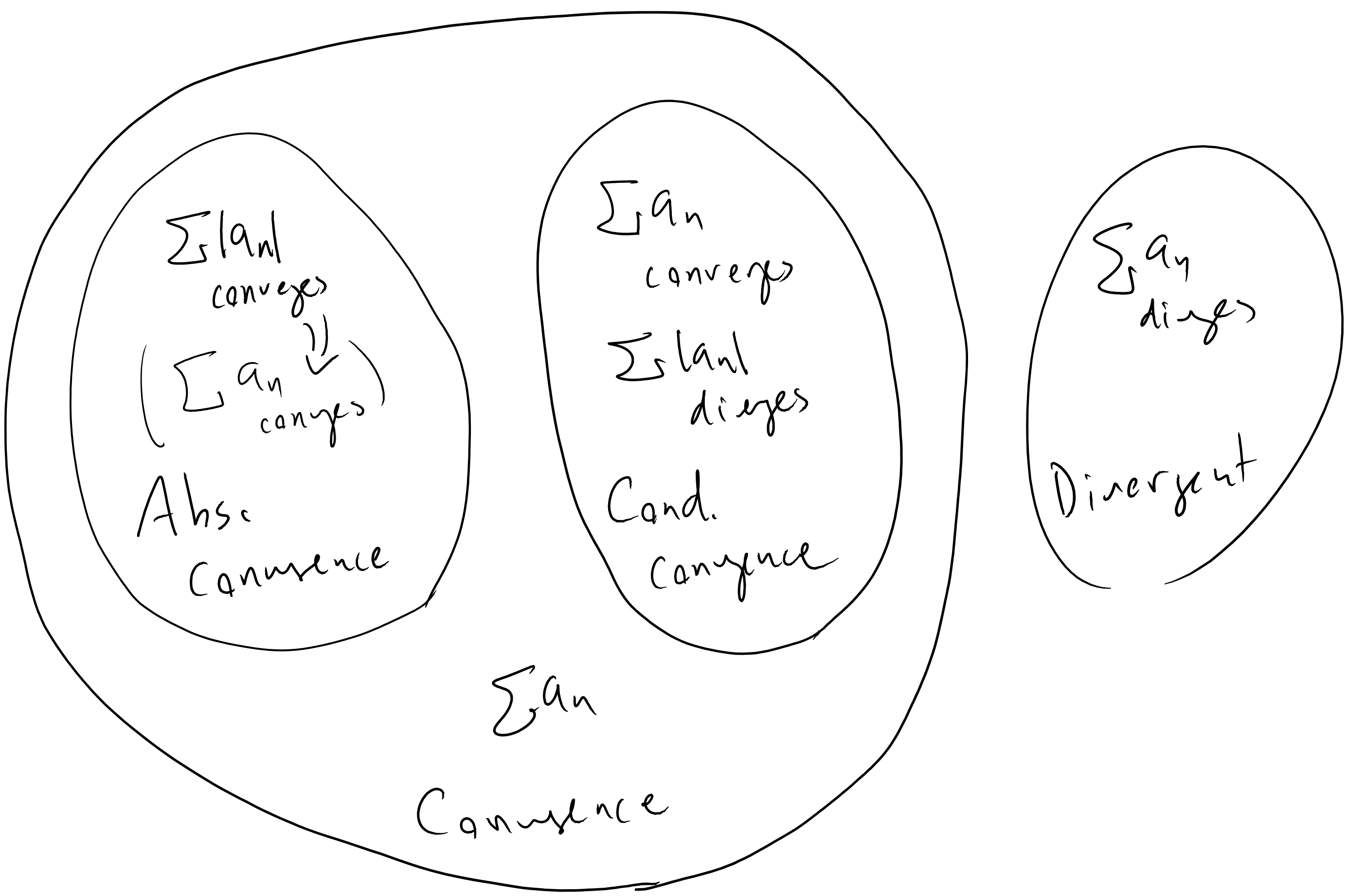
Wednesday, November 12, 2014 12:21 PM

Topics

Sequences & their limits

9.1 { - L'hopital's rule
- Squeeze/sandwich

Series
9.2 - geometric series
9.3, 9.4 - tests for convergence: $\int$ test, ratio, root, comparison (for positive)
9.6 - alternating series test
9.6 - absolute & conditional convergence
9.7 - Power series (for which values of x does a power series converge?)
9.8 - Taylor series.



$$\sum_{n=1}^{\infty} \frac{4^n n!}{n^n}$$

$$\lim_{n\to\infty} \frac{\left|\dfrac{4^{n+1}(n+1)!}{(n+1)^{n+1}}\right|}{\left|\dfrac{4^n(n!)}{n^n}\right|} = \lim_{n\to\infty} \frac{4^{n+1}(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{4^n n!}$$

$$= \lim_{n\to\infty} \frac{4^{n+1}}{4^n} \cdot \frac{(n+1)!}{n!} \cdot \frac{n^n}{(n+1)^{n+1}}$$

$$= \lim_{n\to\infty} 4 \cdot \cancel{(n+1)} \cdot \frac{n^n}{(n+1)^n \cancel{(n+1)}} = \lim_{n\to\infty} 4 \cdot \left(\frac{n}{n+1}\right)^n$$

$$= 4 \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = 4 \cdot \frac{1}{e}$$

$$\lim_{x\to\infty} \left(\frac{x}{x+1}\right)^x = L$$

$$\lim_{x\to\infty} \ln\left(\frac{x}{x+1}\right)^x = \ln L$$

$$\lim_{x\to\infty} x \ln\left(\frac{x}{x+1}\right) = \lim_{x\to\infty} \frac{\ln\left(\frac{x}{x+1}\right)}{1/x}$$

$$= \lim_{x\to\infty} \frac{1/\left(\frac{x}{x+1}\right) \cdot \left(\frac{(x+1)\cdot 1 - x(1)}{(x+1)^2}\right)}{\left(-1/x^2\right)}$$

$$= \lim_{x\to\infty} \left(\cancel{\frac{x+1}{x}}\right)\left(-\frac{x^{\cancel{2}}}{1}\right)\left(\frac{1}{(x+1)^{\cancel{2}}}\right)$$

$$= \lim_{x\to\infty} -\frac{x}{x+1} = -\lim_{x\to\infty} \frac{x}{x+1} \underset{\text{L'hop}}{=} -\lim_{x\to\infty} \frac{1}{1}$$

$$= -1$$

$\Rightarrow \ln(L) = -1$

$$L = e^{-1} = \frac{1}{e}$$

we found:

$\lim$ of $\frac{a_{n+1}}{a_n} = 4/e > 1$ diverges.

For which values of x does the series

(4c) $\sum_{n=1}^{\infty} \frac{1}{n^2} 5^n (4x-3)^n$ converge?

$$\lim_{n\to\infty} \left| \frac{\frac{1}{(n+1)^2} 5^{n+1} (4x-3)^{n+1}}{\frac{1}{n^2} 5^n (4x-3)^n} \right| = \lim_{n\to\infty} \left| \frac{n^2}{(n+1)^2} 5 (4x-3) \right|$$

$$= \lim_{n\to\infty} \left| \left(\frac{n}{n+1}\right)^2 5(4x-3) \right| = |5(4x-3)|$$

$$= |20x - 15|$$

converges if $|20x-15| < 1$ diverges if $|20x-15| > 1$

$$-1 < 20x - 15 < 1$$

$$14 < 20x < 16$$

$$\frac{14}{20} < x < \frac{16}{20} \qquad \frac{7}{10} < x < \frac{8}{10}$$

converges

diverges if $x > \frac{8}{10}$, $x < \frac{7}{10}$

$\frac{7}{10}$? $\frac{8}{10}$?

$$\left[\frac{7}{10}, \frac{8}{10}\right]$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} 5^n \left(4 \cdot \frac{7}{10} - 3\right)^n = \sum_{n=1}^{\infty} \frac{5^n}{n^2} \left(-\frac{2}{10}\right)^n$$

$$= \sum_{n=1}^{\infty} \left(5 \cdot \left(-\frac{2}{10}\right)\right)^n \cdot \frac{1}{n^2}$$

$$= \sum (-1)^n \cdot \frac{1}{n^2}$$

alt. series $\Rightarrow$ converges.

$\frac{8}{10} \rightsquigarrow \sum_{n=1}^{\infty} \frac{1}{n^2}$ $\int$ test $\rightsquigarrow$ converge

terms $\frac{1}{n^2}$ monotonically decreasing terms $\frac{1}{n^2}$ monotonically decreasing

and $\lim_{n\to\infty}\frac{1}{n^2}=0$