Lecture 32: Exam 3 Review

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example (#2, from HW)

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{5}{4^n} = (-1)^2 \frac{5}{4^1} + (-1)^3 \frac{5}{4^2} + (-1)^4 \frac{5}{4^3} + \cdots$$

$\cdot(-1)\cdot\frac{1}{4}$ $\cdot(-1)\cdot\frac{1}{4}$

$a + ar + ar^2 + \cdots$

$\cdot r$ $\cdot r$

$= \frac{a}{1-r}$

$a = \text{first term} = (-1)^2 \frac{5}{4^1} = \frac{5}{4}$

$r = (-1)\cdot\frac{1}{4} = -\frac{1}{4}$

$$\text{sum} = \frac{a}{1-r} = \frac{5/4}{1-(-\frac{1}{4})}$$

$$= \frac{5}{4}\cdot\frac{1}{1+\frac{1}{4}}$$

$$= 1$$

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$$

check absolute convergence

$$\rightsquigarrow \sum \left|\frac{\sin(n)}{n^2}\right| = \sum \frac{|\sin(n)|}{n^2}$$

$|\sin(n)| < 1$ $\qquad \frac{|\sin(n)|}{n^2} < \frac{1}{n^2}$

n^2

$\sum \frac{1}{n^2}$ converges $\Rightarrow \sum \frac{|\sin(n)|}{n^2}$ converges

$\Rightarrow \sum \frac{\sin(n)}{n^2}$ converges absolutely

$\Rightarrow$ it converges.

<u>Taylor expansion</u>

$f(x) = e^{2x}$ about $x = 2$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = \frac{f^{(0)}(a)}{0!}(x-a)^0 + \frac{f^{(1)}(a)}{1!}(x-a)^1 + \dots$$

$0! = 1$

$$= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2$$

$$+ \frac{f'''(a)}{3!}(x-a)^3 + \frac{f''''(a)}{4!}(x-a)^4 + \dots$$

first 3 terms

$$T_3(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$$

$f(x) = e^{2x}$ $\quad a = 2$ $\quad\quad f(2) = e^4$

$f(x) = e^{2x}$ $\quad a = 2$ $\qquad f(2) = e^{4}$

$f(2) + f'(2)(x-2) + \frac{1}{2} f''(2)(x-2)^2$

$f'(x) = 2e^{2x}$ $\qquad f''(x) = 4e^{2x}$

$f'(2) = 2e^4$ $\qquad f''(2) = 4e^4$

$$e^4 + 2e^4(x-2) + \frac{1}{2} \cdot 4e^4 (x-2)^2$$

$f(x) = \sin x$ about $x = 0$ 5 terms.

$$\cancel{f(0)} + f'(0)(x-0) + \cancel{\frac{f''(0)}{2!}(x-0)^2} + \frac{f'''(0)}{3!}x^3 + \cancel{\frac{f''''(0)}{4!}}x^4$$

$2! = 2 \cdot 1 = 2$

$3! = 3 \cdot 2 \cdot 1 = 6$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$5! = 120$

$6! = 720$

$f'(x) = \cos x$ $\qquad f'(0) = 1$

$f''(x) = -\sin x$ $\qquad f''(0) = 0$

$f'''(x) = -\cos x$ $\qquad f'''(0) = -1$

$f''''(x) = \sin x$ $\qquad f''''(0) = 0$

$f(x) = \sin x$ $\qquad f(0) = 0$

$$T_5(x) = 1 \cdot x - \frac{1}{3!}x^3 = x - \frac{1}{6}x^3$$

whole thing:

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \frac{1}{11!}x^{11}$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 - \frac{1}{10!}x^{10}$$

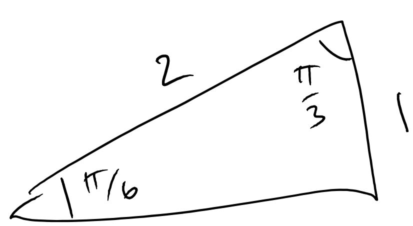
$\sin x$ about $x = \frac{\pi}{3}$ first 3 trms

$$f\left(\frac{\pi}{3}\right) + f'\left(\frac{\pi}{3}\right)\left(x - \frac{\pi}{3}\right) + \frac{f''\left(\frac{\pi}{3}\right)}{2!}\left(x - \frac{\pi}{3}\right)^2$$

$f(x) = \sin x \qquad f\left(\frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$f'(x) = \cos x \qquad f'\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

$f''(x) = -\sin x \qquad f''\left(\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$



$\sqrt{3}$

$-\frac{\sqrt{3}}{2}$

$$f\left(\frac{\pi}{3}\right) + f'\left(\frac{\pi}{3}\right)\left(x-\frac{\pi}{3}\right) + \frac{f''\left(\frac{\pi}{3}\right)}{2!}\left(x-\frac{\pi}{3}\right)^2$$

$\frac{\sqrt{3}}{2}$ $\quad$ $\frac{1}{2}$

$$\frac{\sqrt{3}}{2} + \frac{1}{2}\left(x-\frac{\pi}{3}\right) - \frac{\sqrt{3}}{2}\cdot\frac{1}{2}\left(x-\frac{\pi}{3}\right)^2$$