Lecture 24: Sequences... why?

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Exponential growth

$P(t)$

$\frac{d}{dt}P(t) = kP(t)$ k = rate

$P(0) = 10$

$P(t) \approx 10$ $P(t) = 10 + at$

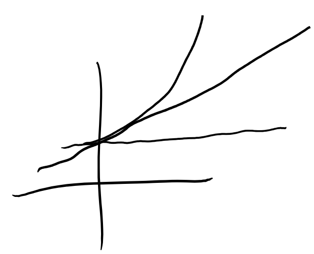
$P'(t) = kP(t)$?

$a = k(10 + at)$

$a = k10 + k \cdot a \cdot t$ want good approx at small t.

$a \approx 10k + \text{small}$ t small

$\underline{a = 10k}$



$P(t) \approx 10 + (10k)t$

$P(t) \approx 10 + 10kt + a_2 t^2$

$$P'(t) = kP(t)$$

$$10k + 2a_2t = k(10 + 10kt + a_2t^2)$$

$$\cancel{10k} + 2a_2t = \cancel{10k} + 10k^2t + a_2kt^2$$

$$2a_2t = 10k^2t + a_2kt^2$$

$$2a_2 = 10k^2 + \underbrace{a_2kt}_{\text{small}}$$

$$a_2 = 5k^2$$

$$P(t) \approx 10 + (10k)t + (5k^2)t^2 + \cdots$$

$$\frac{10}{0!} + \left(\frac{10k}{1}\right)t + \left(\frac{10k^2}{2\cdot 1}\right)t^2 + \left(\frac{10k^3}{3\cdot 2\cdot 1}\right)t^3 +$$

$$\left(\frac{10k^4}{4\cdot 3\cdot 2\cdot 1}\right)t^4 + \cdots$$

$$P(t) = \sum_{n=0}^{\infty} 10\cdot\frac{k^n t^n}{n!} = 10e^{kt}$$

"Taylor Series"
a type of Power series.

$$\frac{d}{dt}P(t) = P(t)\left(100 - P(t)\right) + \sin t$$

$p(t) = c_0 + a_1 t + a_2 t^2 \ldots$

(2nd course on numerical analysis)
Diff eqns

Standard Limits

Indeterminate forms : you can't "get" these.

$\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, \infty \cdot 0, \infty^0, 0^0, 1^\infty$

$\boxed{\text{L'hopital}}$

$\lim_{x\to\infty} x \cdot \frac{1}{x} = 1$

$\lim_{x\to\infty} x^2 \cdot \frac{1}{x} = \infty$

$\lim_{x\to\infty} x \cdot \frac{1}{x^2} = 0$

$\lim_{x\to\infty} (\underbrace{e^x}_{\infty})^{\overbrace{1/x}^{0}} = \lim_{x\to\infty} e^1 = e$

$\lim_{x\to\infty} (e^{-x})^{\overbrace{1/x}^{0}} = e^{-1}$

$$\lim_{x\to\infty} \left(e^{\underbrace{\cdot}_{0}}\right) = e^{\cdot}$$

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for forms other than $\frac{0}{0}$ or $\frac{\infty}{\infty}$, need to rewrite as a fraction.

$$\lim_{x\to\infty} \left(\underbrace{1 - \frac{1}{x}}_{\to 1}\right)^{x \to \infty}$$

$0^0 \quad \infty^0 \quad 1^\infty$

strategy take logs.

$$1^\infty$$

$$L = \lim_{x\to\infty} \left(1 - \frac{1}{x}\right)^x$$

$$\ln L = \ln \lim_{x\to\infty} \left(1 - \frac{1}{x}\right)^x = \lim_{x\to\infty} \ln\left(1 - \frac{1}{x}\right)^x$$

$$= \lim_{x\to\infty} x \cdot \ln\left(1 - \frac{1}{x}\right)$$

" $\infty \cdot 0$ "

$$\lim_{x\to\infty} \frac{\ln\left(1 - \frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$$

$0^0, \infty^0, 1^\infty$

$0 \cdot \infty$, ∞^0 , 1^∞

$\downarrow \ln$

$$\lim_{x\to\infty} \frac{x}{\left(1/\ln(1-\frac{1}{x})\right)}$$

$0 \cdot \infty$

$$A \cdot B = \frac{B}{\left(\frac{1}{A}\right)} = \frac{A}{\left(\frac{1}{B}\right)}$$

$\infty - \infty$ common denominator

$\frac{0}{0}$, $\frac{\infty}{\infty}$

$$\ln L = \lim_{x\to\infty} \frac{\ln(1-\frac{1}{x})}{\left(\frac{1}{x}\right)} = \lim_{x\to\infty} \frac{\left(1/(1-\frac{1}{x})\right)\cdot\frac{1}{x^2}}{\left(-1/x^2\right)}$$

$$= \lim_{x\to\infty} \frac{-1}{\left(1-\frac{1}{x}\right)} = -1$$

$$\ln L = -1$$

$$L = e^{-1} = \frac{1}{e}$$

$$a_n = \frac{2^n}{n!}$$

$a_1 = \frac{2^1}{1!} = \frac{2}{1}$ $\qquad a_2 = \frac{2 \cdot 2}{2!} = \frac{2 \cdot 2}{2 \cdot 1} = \frac{4}{2}$

$$a_6 = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot \boxed{2 \cdot 2}}{6 \cdot 5 \cdot 4 \cdot 3 \cdot \boxed{2 \cdot 1}} \le \frac{2 \cdot 2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3 \cdot 3} \boxed{\frac{2 \cdot 2}{2 \cdot 1}}$$

$$a_{100} = \quad \le \frac{2^{100-2}}{3^{100-2}} \boxed{} \quad \to 0$$

$$\left(\frac{2}{3}\right)^{\text{big}} \to 0$$

$$\lim_{n \to \infty} \frac{2^n}{n!} = 0$$