Lecture 21: exam2 review

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Review Sheet

1 iii) $\int \sin^4 x\,dx = \int (\sin^2 x)^2\,dx$

$\frac{1-\cos 2x}{2} = \sin^2 x$ $\quad \cos^2 x = \frac{1+\cos 2x}{2}$

$$\int \left(\frac{1-\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int (1 - 2\cos 2x + \cos^2 2x)\,dx$$

$$= \frac{1}{4}\int 1\,dx - \frac{2}{4}\int \cos 2x\,dx + \frac{1}{4}\int \cos^2 2x\,dx$$

$u = 2x$

$$= \frac{1}{4}x - \frac{1}{2}\cdot\frac{1}{2}\sin 2x + \frac{1}{4}\int \frac{1+\cos[2(2x)]}{2}\,dx$$

$$= \frac{1}{4}x - \frac{1}{4}\sin 2x + \frac{1}{8}\int (1+\cos 4x)\,dx$$

$$= \frac{1}{4}x - \frac{1}{4}\sin 2x + \frac{1}{8}x + \frac{1}{8}\cdot\frac{1}{4}\sin 4x + C$$

11 xi) $\int \sec^3 x\,dx$ → parts $u = \sec x$, $dv = \sec^2 x$

→ switch $\sec^2 x \to \tan^2 x + 1$

$$\int \sec^3 x\,dx = \sec x \tan x - \int \sec x \tan^2 x\,dx$$

$u = \sec x$ $\quad du = \sec x \tan x\,dx$

$dv = \sec^2 x\,dx$ $\quad v = \tan x$

$$\sec x \tan x - \int \sec x(\sec^2 x - 1)\,dx$$

$$\int \sec^3 x\,dx = \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$$

$$2\int \sec^3 x\,dx = \sec x \tan x + \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x\,dx = \frac{\sec x \tan x + \ln|\sec x + \tan x|}{2} + C$$

$\int \sec^3 x\,dx$ → parts: $= \sec x \tan x - \int \sec x \tan^2 x\,dx$

→ trig sub: $= \int \sec x(1 + \tan^2 x)\,dx$

$$= \int \sec x \, dx + \int \sec x \tan^2 x \, dx$$

$$\int \sec^3 x \, dx = \sec x \tan x - \int \sec x \tan^2 x \, dx$$

$$\int \sec^3 x \, dx = \int \sec x \, dx + \int \sec x \tan^2 x \, dx$$

$$2 \int \sec^3 x \, dx = \sec x \tan x + \ln|\sec x + \tan x|$$

(viii) $\displaystyle\int \frac{x^4 + x + x^2}{x^2 + 1} \, dx$

$$\begin{array}{r|l} & x^2 \text{ rem } x \\ \hline x^2+1 & x^4 + 0x^3 + x^2 + x + 0 \\ & \underline{x^4 + 0x^3 + x^2} \\ & 0 + x + 0 \end{array}$$

$x^2 + 0x + 1$ times x^2

$$\frac{x^4 + x + x^2}{x^2 + 1} = x^2 + \frac{x}{x^2 + 1}$$

$$\frac{x^3+x}{x^2+1} = x \cdot \frac{}{x^2+1}$$

$$\int x^2\,dx + \int \frac{x}{x^2+1}\,dx$$

$$u = x^2+1$$

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1 ix) $$\int \frac{x^2-4}{(x^2+1)x}\,dx$$

$$\frac{x^2-4}{(x^2+1)x} = \frac{Ax+B}{x^2+1} + \frac{C}{x}$$

$$x^2-4 = x(Ax+B) + C(x^2+1)$$

$$x^2-4 = x^2(A+C) + x(B) + C$$

$$B=0 \qquad C=-4 \qquad A=5$$

x^2-4 $\quad$ $5x$ $\quad$ 4 $\quad$ $?$

$$\frac{x^2-4}{x(x^2+1)} = \frac{5x}{x^2+1} - \frac{4}{x} \quad ?$$