Lecture 20: overview, intro to limits

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$$\frac{1}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$$

mult. both sides by $(x-1)(x+2)(x-3)$

$$1 = A(x+2)(x-3) + B(x-1)(x-3) + C(x-1)(x+2)$$

$$1 = x^2(A+B+C) + x(-A-4B+C) + (-6A+3B-2C)$$

$$\int x^5 e^x dx$$

$$\int x^n e^x dx = x^n e^x - n\int e^x x^{n-1} dx$$

$u = x^n$ $du = nx^{n-1}dx$

$dv = e^x dx$ $v = e^x$

$$\int x^5 e^x dx = x^5 e^x - 5\int x^4 e^x dx$$

$x^4 e^x - 4\int$ ..

$$x^5 e^x - 5\left(x^4 e^x - 4\left(x^3 e^x - 3\left(x^2 e^x - 2\left(x e^x - (e^x)\right)\right)\right)\right)$$

$+C$

$$\int \frac{dx}{x(x^2+1)^2}$$

$$\frac{1}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

clear den's: (mult. by $x(x^2+1)^2$)

$$A(x^2+1)^2 + (Bx+C)(x)(x^2+1) + (Dx+E)x$$

$$A(x^4+2x^2+1) + (Bx+C)(x^3+x) + (Dx^2+Ex)$$

$$Ax^4+2Ax^2+A+Bx^4+Bx^2+Cx^3+Cx+Dx^2+Ex$$

$$x^4(A+B) + x^3(C) + x^2(2A+B+D) +$$

$$x(C+E) + (A) = 1$$

$A = 1$ $\quad C = 0$ $\quad E = 0$

$B = -1$ $\quad 2A + B = -D$

$2 - 1$ $\quad D = -1$

$$\frac{1}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

$$= \frac{1}{x} + \frac{-x}{x^2+1} + \frac{-x}{(x^2+1)^2}$$

$$= \frac{1}{x} - \frac{x}{x^2+1} - \frac{x}{(x^2+1)^2}$$

Topics

- Parts
- trig functions
- trig subst.
- Improper Integrals
- Partial fractions

$$\int 4x^4 - 3x^2 + 2 \; dx = \int (-4x^2 - 1) + \frac{3}{1-x^2} \, dx$$

$$\int \frac{4x^4-3x^2+2}{1-x^2}\,dx = \int(-4x^2-1)+\frac{5}{1-x^2}\,dx$$

$$\begin{array}{r l}
 & -4x^2 \quad -1 \quad \text{rem } 3 \\
-x^2+0x+1 \,\big) & 4x^4+0x^3-3x^2+0x+2 \\
 & \underline{4x^4+0x^3-4x^2} \\
 & \qquad\qquad x^2+0x+2 \\
 & \qquad\qquad \underline{x^2+0x-1} \\
 & \qquad\qquad\qquad\quad 3
\end{array}$$

Work : F·d

$\frac{1}{x^2}$.

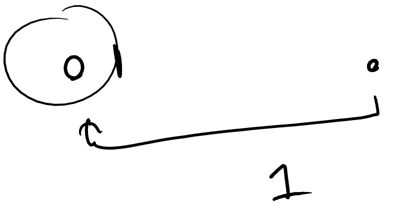
$$\int_1^\infty \frac{1}{x^2}dx = \lim_{t\to\infty}\int_1^t \frac{1}{x^2}dx$$

$$= \lim_{t\to\infty}\left[-\frac{1}{x}\right]_1^t$$

$$= \lim_{t \to \infty} \left(-\frac{1}{t} - \left(-\frac{1}{1}\right)\right)$$

$$= \lim_{t \to \infty} \left(1 - \cancelto{0}{\frac{1}{t}} \right)$$

$$= 1.$$



$$\int_1^0 F\,dx = \int_1^0 \frac{1}{x^2}\,dx = \lim_{t \to 0} \int_1^t \frac{1}{x^2}\,dx$$

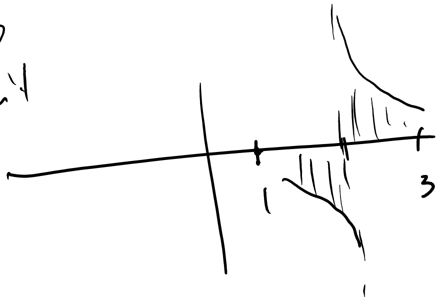
$$= \lim_{t \to 0} \left[-\frac{1}{x} \right]_1^t = \lim_{t \to 0} \left[-\frac{1}{t} + \frac{1}{1} \right]$$

$$= \lim_{t \to 0^+} 1 - \frac{1}{t}$$

ewg7 $\to \infty$!

$$\int_1^3 \frac{1}{\sqrt[3]{x-2}}\,dx = \int_1^2 \frac{1}{\sqrt[3]{x-2}}\,dx + \int_2^3 \frac{1}{\sqrt[3]{x-2}}\,dx$$

domain doesn't contain 2!



$$\lim_{t\to 2^-}\int_1^t \frac{1}{\sqrt[3]{x-2}}\,dx + \lim_{t\to 2^+}\int_t^3 \frac{1}{\sqrt[3]{x-2}}\,dx$$

$$\lim_{t\to 2^-}\left[\frac{3}{2}(x-2)^{2/3}\right]_1^t + \lim_{t\to 2^+}\left[\frac{3}{2}(x-2)^{2/3}\right]_t^3$$

$$\lim_{t\to 2^-}\left(\frac{3}{2}\cancelto{0}{(t-2)^{2/3}} - \frac{3}{2}(1-2)^{2/3}\right] + \lim_{t\to 2^+}\left(\frac{3}{2}(3-2)^{3} - \frac{3}{2}\cancelto{0}{(t-2)^{2/3}}\right)$$

$$-\frac{3}{2}(1-2)^{2/3} + \frac{3}{2}(3-2)^{2/3} = 0.$$

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f' $\qquad$ $\dot{f}$ $\qquad$ $\dfrac{dy}{dx}$

Newton $\qquad$ Liebniz - Infinitesimals

Newton
~ Limits

Liebniz - infinitesimals

$\frac{d}{dx}(x^2)$ let dx be infinitesimaly small

$$\frac{d(x^2)}{dx} \qquad \frac{(x+dx)^2-x^2}{dx}$$

$$= \frac{x^2+2x\,dx+dx^2-x^2}{dx}$$

$$= \frac{2x\,dx+dx^2}{dx}$$

$$= 2x+dx \approx 2x$$

Limits

sequences:

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

$a_n = \frac{1}{n}$ what does it mean to say

$$\lim_{n\to\infty} a_n = 0 \;?$$

$$\underset{x}{\underset{\shortparallel}{.99999\ldots\ldots}} = 1$$

$$9x = 9$$

$$x = 1$$

$$10x - x = 9$$

$$9.999\ldots\ldots - .999\ldots = 9$$

$$\underset{x}{\underset{\shortparallel}{.333\ldots}}$$

$$10x - x = 3 \;\longrightarrow\; 9x = 3$$

$$x = \frac{3}{9} = \frac{1}{3}.$$

$$3.333\ldots - .33\ldots = 3$$

$$a_n = .\underbrace{999 \quad 9}_{n \text{ times}}$$

$$\lim_{n\to\infty} a_n = 1$$

Definition:

$\lim_{n\to\infty} a_n = L$ means for any positive number ε

there exists an integer N such that

$|a_n - L| < \varepsilon$ whenever $n \geq N$.

why does $\lim_{n\to\infty} \frac{1}{n} = 0$?

given any $\varepsilon > 0$, want to choose N

pick $N > \frac{1}{\varepsilon}$. if $n \geq N$ then

$n \geq N > \frac{1}{\varepsilon}$ $\qquad$ $n > \frac{1}{\varepsilon}$ & so

$\varepsilon > \frac{1}{n}$

$\Rightarrow \left| \frac{1}{n} - 0 \right| = \frac{1}{n} < \varepsilon$