Lecture 2 (recitation): Integration Practice

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Basic Rules

$$\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int c f(x)\,dx = c\int f(x)\,dx$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C \quad \text{unless } n=-1$$

$$\int x^{-1}\,dx = \ln|x| + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \tan x\,dx$$

Practice

1. $\int 2x\,dx$ 2. $\int (1+\cos x)\,dx$

3. $\int \sin 2x\,dx$ 4. $\int (1-3x^{-4}+3\sqrt{x}+e^{x})\,dx$

1. $2\frac{1}{1+1}x^{1+1}+C = x^2+C$

2. $\int \underset{x^0}{1}\,dx + \int \cos x\,dx = \frac{1}{0+1}x^{0+1} + \sin x + C$

$= x + \sin x + C$

4. $\int(-3x^{-4}+3\sqrt{x}+e^x)dx$

$= -3\int x^{-4}dx + 3\int x^{1/2}dx + \int e^x dx$

$= -3\frac{1}{-4+1}x^{-4+1} + 3\frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1} + e^x + C$

$= x^{-3} + 2x^{3/2} + e^x + C$

3. $\int \sin 2x\,dx = -\frac{1}{2}\cos 2x + C.$

$-\cos ?$ $\quad (-\cos 2x)\cdot\frac{1}{2}$

$\downarrow d/dx$

$$\left(2\sin 2x\right)\cdot\frac{1}{2}=\sin 2x$$

<u>u-substitution</u> (Anti-Chain Rule)

$$\frac{d}{dx}F(g(x))=F'(g(x))\cdot g'(x) \qquad \text{write } f(x)=F'(x)$$

$F(g(x))$ is an anti-derivative of $f(g(x))g'(x)$

$$\int f(\underbrace{g(x)}_{u})\underbrace{g'(x)dx}_{du}=F(g(x))+C$$

Notation: $u=g(x) \qquad du=g'(x)\,dx$

$$\int f(u)\,du=F(u)+C$$

<u>ex:</u> $\displaystyle\int \cot x\,dx=\int\frac{\cos x}{\sin x}dx=\int\frac{1}{\sin x}\cos x\,dx$

$u=\sin x$

$du=\cos x\,dx$

$$\int\frac{1}{u}du=\ln|u|+C=\boxed{\ln|\sin x|+C}$$