Lecture 17: trig and some partial fractions

Wednesday, October 1, 2014 12:38 PM

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{du}{u} = -\ln|u| + C$$

$u = \cos x$

$du = -\sin x\,dx$

$= -\ln|\cos x| + C$

$= \ln|\cos x|^{-1} + C$

$= \ln|\sec x| + C$

$\int \tan^2 x\,dx$

$\int \tan^3 x\,dx$

$$\int (\sec^2 x - 1)\,dx$$

$$\int \sec^2 x\,dx - \int dx$$

$$= \tan x - x + C$$

$$\int \tan x(\sec^2 x - 1)\,dx$$

$$= \int \tan x \sec^2 x\,dx - \int \tan x\,dx$$

$u = \tan x$

$u = \sec x$

$$\int u\,du - \ln|\sec x|$$

$$\frac{1}{2}\sec^2 x - \ln|\sec x| + C$$

$$\frac{1}{2}\tan^2 x - \ln|\sec x| + C$$

$\sec^2 x = \tan^2 x + 1$ (same derivative)

Partial Fractions

Start pt: $\frac{\text{Liner}}{\text{Quadratic}} = \frac{2x-3}{x^2-1}$

WANT $\frac{\text{LIN}}{\text{QUAD}} = \frac{\text{CONST}}{\text{LIN}} + \frac{\text{CONST}}{\text{LIN}}$

↖ ↗ factrs of Quad.

$$\frac{2x-3}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{x+1}{x+1}\frac{A}{x-1} + \frac{B}{x+1}\frac{x-1}{x-1}$$

$(x+1)(x-1)$

$$\frac{2x-3}{x^2-1} = \frac{Ax+A+Bx-B}{(x+1)(x-1)}$$

$$\frac{2x-3}{x^2-1} = \frac{(A+B)x + (A-B)}{x^2-1}$$

$$2 = A+B$$

$$-3 = A-B$$

$$A = -\frac{1}{2}$$

$$-1 = 2A \qquad A = -\frac{1}{2}$$

$$B - \frac{1}{2} = 2$$

$$B = 5/2$$

$$\frac{2x-3}{x^2-1} = \frac{\left(-\frac{1}{2}\right)}{x-1} + \frac{(5/2)}{x+1}$$