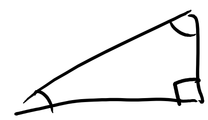
Lecture 15: trig, trig, trig

Thursday, September 25, 2014 12:29 PM

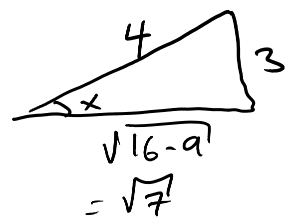
all $\angle$'s today are "right Δ angles" (between 0 & $\pi/2$)



$\sin x = \frac{3}{4}$

what is $\tan x = 3/\sqrt{7}$

what is $\sec x = 4/\sqrt{7}$



$\tan\left(\arcsin \frac{3}{4}\right)$

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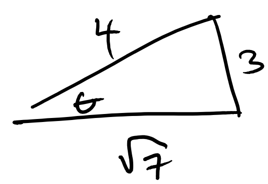
$\sin\theta = \frac{3}{4}$

$\theta = \arcsin \frac{3}{4}$

$\tan\theta = 3/\sqrt{7}$

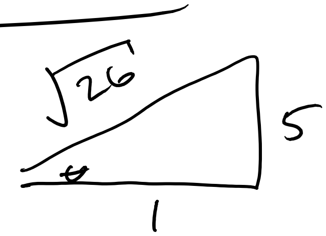


what is $\tan\left(\arcsin \frac{3}{4}\right)$

if $\sin\theta = \frac{3}{4}$, what is $\tan(\theta)$

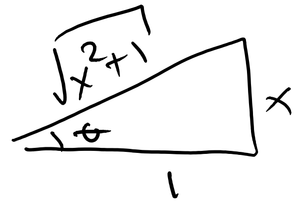
$\sec(\arctan 5) =$

$\tan\theta = 5$, what is $\sec\theta$



$\tan\theta = 5$, what is $\sec\theta$ $= \sqrt{26}$

$\sin(\arctan x)$



$\tan\theta = x \qquad \sin\theta = \dfrac{x}{\sqrt{x^2+1}}$

$\cos(\arcsin x) = \sqrt{1-x^2}$

$\int \arcsin x \, dx = \int u \cos u \, du$

$\arcsin(\sin x) = x$

$u = \arcsin x$

$\sin(\arcsin x) = x$

$\sin u = x$

$\cos u \, du = dx$

$\int v \, dw = vw - \int w \, dv$

$\int u \cos u \, du = u \sin u - \int \sin u \, du$

$v = u \qquad dv = du$

$dw = \cos u \, du \qquad w = \sin u$

$= u \sin u + \cos u + C$

$= (\arcsin x)\, x + \cos(\arcsin x) + C$

$= x \arcsin x + \sqrt{1-x^2} + C$

$$\int \sqrt{1-x^2}\,dx = -\int \sqrt{\sin^2 u}\ \sin u\,du$$

$\sin^2 u + \cos^2 u = 1$

$\cos u = x$

$(u = \arccos x)$

$1 - \sin^2 u = \cos^2 u$

$1 - \cos^2 u = \sin^2 u$

$-\sin u\,du = dx$

$\sin^2 u = \dfrac{1-\cos 2u}{2}$

$\cos^2 u = \dfrac{1+\cos 2u}{2}$

$$-\int \sin^2 u\,du$$

$$= -\int \left(\frac{1}{2} - \frac{1}{2}\cos 2u\right) du$$

$$= -\frac{1}{2}u + \frac{1}{2}\int \cos 2u\,du$$

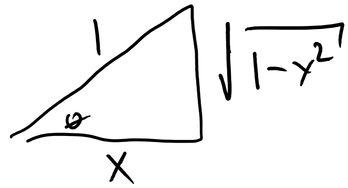
$$-\frac{1}{2}u + \frac{1}{4}\sin 2u + C$$

$$-\frac{1}{2}\arccos x + \frac{1}{4}\sin(2(\arccos x)) + C$$

$\sin 2x = 2\sin x \cos x$

$$-\frac{1}{2}\arccos x + \frac{1}{4}\,2\sin(\arccos x)\cos(\arccos x) + C$$

$$-\frac{1}{2}\arccos x + \frac{1}{2}\sin(\arccos x)\ x + C$$



$\sin\theta = \sqrt{1-x^2}$

$$= -\frac{1}{2}\arccos x + \frac{1}{2}\sqrt{1-x^2}\, x + C$$

$$\int \frac{1}{x^2\sqrt{x^2-1}}\,dx$$

$\sin^2 u + \cos^2 u = 1$

$1 - \cos^2 u = \sin^2 u$

$1 - \sin^2 u = \cos^2 u$

$x = \sec u$

$dx = \sec u \tan u\, du$

$\frac{\sin^2 u}{\cos^2 u} + 1 = \frac{1}{\cos^2 u}$

$$\int \frac{1}{\sec^2 u\sqrt{\sec^2 u - 1}} \sec u \tan u\, du$$

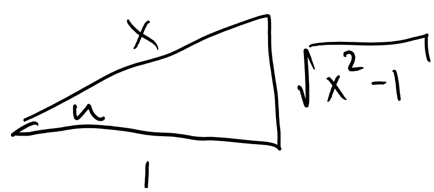
$\tan^2 u + 1 = \sec^2 u$

$\tan^2 u = \sec^2 u - 1$

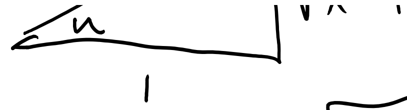
$$= \int \frac{\cancel{\sec u}\,\cancel{\tan u}}{\sec^{\cancel{2}} u\,\cancel{\sqrt{\tan^2 u}}}\,du$$

$$= \int \frac{1}{\sec u}\,du = \int \cos u\, du = \sin u + C$$

$\sec u = x$



$\sqrt{x^2-1}$



$$= \frac{\sqrt{x^2-1}}{x} + C$$

$$\sin u = \frac{\sqrt{x^2-1}}{x}$$

$$= \sqrt{1-x^{-2}} \xrightarrow{d/dx} \frac{1}{2\sqrt{1-x^{-2}}} \cdot 2x^{-3}$$

$$= \frac{x^{-3}}{\sqrt{1-x^{-2}}} .$$

$$= \frac{1}{x^2 \cdot x\sqrt{1-x^{-2}}} = \frac{1}{x^2\sqrt{x^2-1}}$$

$$\int_{x=0}^{x=2} \frac{dx}{8+2x^2} = \int_{u=0}^{u=1} \frac{2du}{8+8u^2} = \frac{1}{4}\int_{u=0}^{u=1} \frac{du}{1+u^2}$$

aiming for $\frac{1}{1+x^2}$

$$2x^2 \rightsquigarrow 8u^2$$

$$x^2 = 4u^2$$

$$\boxed{x = 2u}$$

$$dx = 2du$$

$$u = \tan v$$

$$du = \sec^2 v\, dv$$

$$\frac{1}{4}\int_{v=0}^{v=\pi/4} \frac{\sec^2 v}{1+\tan^2 v}\, dv$$

$$= \frac{1}{4}\int_0^{\pi/4} 1\, dv$$

$$= \frac{1}{4}\left[v\right]_0^{\pi/4} = \frac{\pi}{16}.$$

$$\boxed{e^{i\pi} = -1}$$

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \cdots$$

$$= 1 + \frac{1}{1}x^1 + \frac{1}{2\cdot 1}x^2 + \frac{1}{3\cdot 2\cdot 1}x^3 + \frac{1}{4\cdot 3\cdot 2\cdot 1}x^4 + \cdots$$

$$e^x = a_0 + a_1 x + a_2 x^2 + \cdots \qquad a_0 = e^0 = 1$$

$$\| \qquad \| \qquad \|$$

$$(e^x)' = a_1 + 2a_2 x + 3a_3 x^2 + \cdots$$

$$\frac{e^{i\theta} - e^{-i\theta}}{2} = i\sin\theta$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\frac{e^{i\theta} + e^{-i\theta}}{2} = \cos\theta \qquad e^{i\pi} = -1$$

$$\frac{\quad}{2} = \cos\theta \qquad e^{i\pi} = -1$$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

$$e^{2i\theta} = e^{i\theta}e^{i\theta} = (\cos\theta + i\sin\theta)(\cos\theta + i\sin\theta)$$

$$\shortparallel$$

$$\cos 2\theta + i\sin 2\theta \qquad \cos^2\theta - \sin^2\theta + 2i\sin\theta\cos\theta$$

$$\boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta}$$

$$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$$