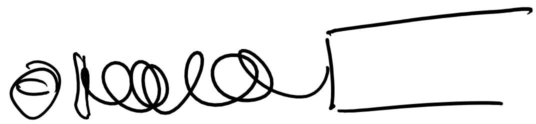
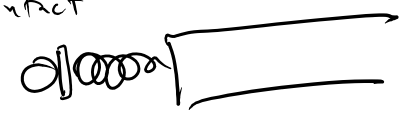
Lecture 12: various things

Thursday, September 11, 2014 12:31 PM

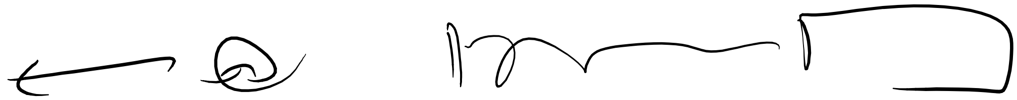
Q1:



contact



let it go!



how fast?

ball weighs 1 standard unit M

sprng exert a force prop to disp. from rest.

$F = -\text{disp.} = L$

push 3 units to right

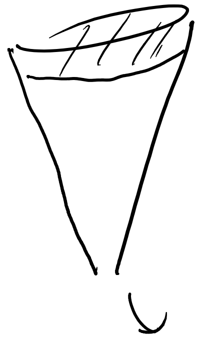
Work: energy done on object = force · dist

$$\int_{x=a}^{x=b} F(x)\,dx$$

$$\text{ball's energy} = \int_0^3 F(x)\,dx = \int_0^3 x\,dx = \frac{1}{2}x^2\Big]_0^3 = \frac{9}{2}$$

$$E = \frac{1}{2}\underset{\substack{\uparrow \\ 1}}{m} v^2 = \frac{1}{2} v^2 = \frac{9}{2}$$

$v = 3$ velocity to left.



Strategy for dealing w/ $\sqrt{\quad}$'s in arclengths, surface area

typical situations:

$$\sqrt{x^2 + \frac{1}{2} + \frac{1}{16x^2}} = \left(\text{some stuff}\right)\sqrt{\text{POLY}}$$

easier

$$\sqrt{x^2+2x+1} = \sqrt{(x+1)^2}$$

$$\sqrt{x^2+4x+4} = \sqrt{(x+2)^2}$$

$$\sqrt{x^4+4x^2+4} = \sqrt{(x^2+2)^2}$$

$$\sqrt{x^2+\frac{1}{2}+\frac{1}{16x^2}} = \sqrt{\frac{16x^2}{16x^2}\left(x^2+\frac{1}{2}+\frac{1}{16x^2}\right)}$$

$$= \sqrt{\frac{1}{16x^2}\left(16x^4+8x^2+1\right)}$$

$$= \sqrt{\frac{1}{16x^2}\left(4x^2+1\right)^2}$$

$$= \frac{1}{4}\frac{1}{x}\left(4x^2+1\right)$$

$$\int \sin x \cos^3 x \, dx = -\int u^3 \, du \ \ldots$$

$u = \cos x$

$du = -\sin x \, dx$

$$\int \sin^2 x \cos^3 x \, dx = \int \sin^2 x \cos x \cos^2 x \, dx$$

"

$2 \quad 1 \quad \cos 2x$

$$\cos^2 x = 1 - \sin^2 x$$

$$\int \sin^2 x \cos x (1 - \sin^2 x)\,dx$$

$$\parallel$$

$$\int \underbrace{\sin^2 x \cos x\,dx}_{u^2\,du} - \int \underbrace{\sin^4 x \cos x\,dx}_{u^4\,du}$$

$u = \sin x$

$du = \cos x\,dx$

$$\int_{x=0}^{x=\pi/2} \cos^2 x \sin x\,dx = -\int_{1=\cos 0}^{\cos \pi/2 = 0} u^2\,du = -\int_1^0 u^2\,du = \frac{1}{3}1^3$$

$u = \cos x$

$du = -\sin x\,dx$

if $x=0$ what is $u=$?

$u = \cos x$

$\cos 0 = u = 1$

$$\int \sin^5 x \cos^5 x\,dx = \int \sin^5 x \cos^4 x \cos x\,dx$$

$$= \int \sin^5 x (\cos^2 x)^2 \cos x\,dx$$

$$= \int \sin^5 x \,(1-\sin^2 x)^2 \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

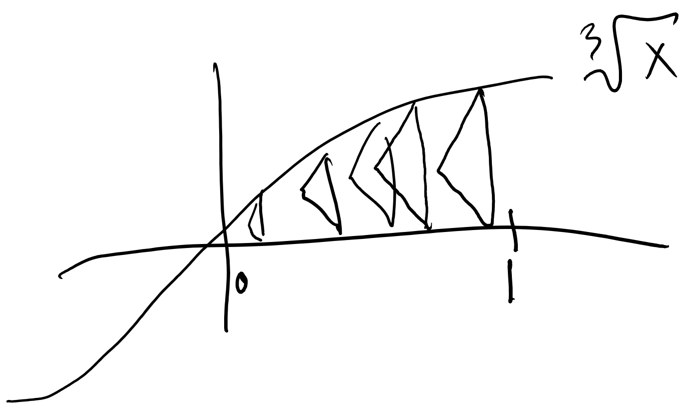
$$= \int u^5 (1-u^2)^2 \, du$$

$$= \int u^5 (1 - 2u^2 + u^4) \, du$$

$$= \int u^5 - 2u^7 + u^9 \, du$$

$$= \frac{1}{6}u^6 - \frac{2}{8}u^8 + \frac{1}{10}u^{10} + C$$

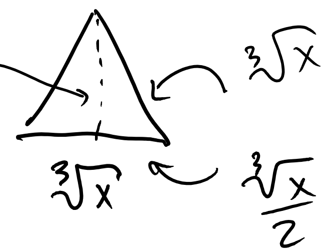
$$= \frac{1}{6}\sin^6 x - \frac{1}{4}\sin^8 x + \frac{1}{10}\sin^{10} x + C$$



equilateral Δ's
cross-sections

$$V = \int_0^1 A(x)\,dx$$

$A(x) =$ area of



$$h^2 + \left(\frac{\sqrt[3]{x}}{2}\right)^2 = \left(\sqrt[3]{x}\right)^2$$

$$h^2 = x^{2/3} - \frac{x^{2/3}}{4}$$

$$h^2 = \frac{3}{4}x^{2/3}$$

$$h = \frac{\sqrt{3}}{2}x^{1/3}$$

$$= \frac{1}{2}b \cdot h = \frac{1}{2}\left(\sqrt[3]{x}\right)\left(\frac{\sqrt{3}}{2}\sqrt[3]{x}\right)$$

$$= \frac{\sqrt{3}}{4}\left(x^{1/3} \cdot x^{1/3}\right) = \frac{\sqrt{3}}{4}x^{2/3}$$

$$V = \int_0^1 A(x)\,dx = \int_0^1 \frac{\sqrt{3}}{4}x^{2/3}\,dx$$

$$= \frac{\sqrt{3}}{4}\left[\frac{3}{5}x^{5/3}\right]_0^1 = \frac{3\sqrt{3}}{20}$$

$$\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx = \int \frac{1}{2}\,dx - \frac{1}{2}\int \cos 2x\,dx$$

$$\frac{1}{2}x - \frac{1}{4}\sin 2x + C$$

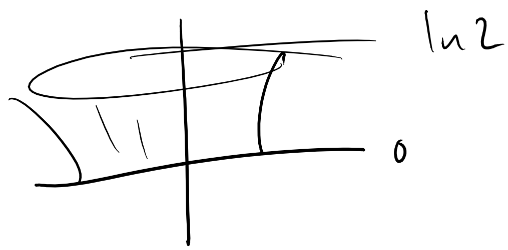
$$\sin^2 x = \frac{-\cos 2x + 1}{2}$$

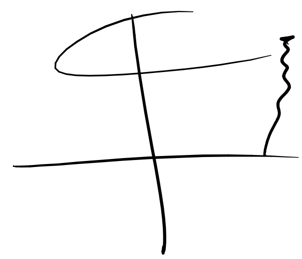
$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

$$= \int x^{-1/2}\sin x^{1/2}\,dx$$

$$x = (e^{y}+e^{-y})/2 \qquad 0 \le y \le \ln 2$$

cosh y





$$SA = \int_0^{\ln 2} 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_0^{\ln 2} 2\pi\, (e^{y}+e^{-y})/2 \sqrt{1+\left(\frac{e^{y}-e^{-y}}{2}\right)^2}\, dy$$

$$\frac{d}{dy}(e^{y}+e^{-y})/2 = \frac{e^{y}-e^{-y}}{2}$$

" sinh y

$$1+\left(\frac{e^{y}-e^{-y}}{\ }\right)^2 = 1+\frac{e^{2y}-2+e^{-2y}}{\ }$$

$$= \frac{e^{2y} + 2 + e^{-2y}}{4}$$

$$= \left(\frac{e^{y} + e^{-y}}{2}\right)^2$$