Lecture 11: practice!

Wednesday, September 10, 2014 12:24 PM

Setup an integral to compute the following volumes

1. region in 1st quadrant between $y = 4$

   i. $y = x^2$

   a) about y-axis

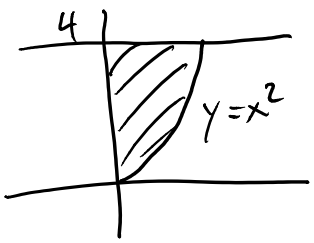
   b) about x-axis

   c) about $x = 2$

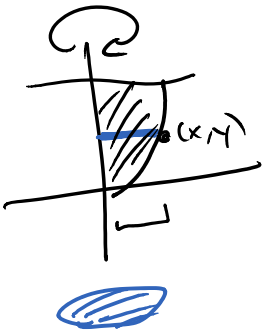
   d) about $y = 4$

   e) about $x = -7$

   f) about $y = -10$



a)



(either works)

disks

disk for each y

$$V = \int_0^4 \pi r^2 \, dy$$

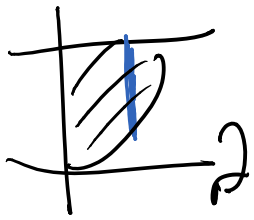
$r = x$ on graph of $y = x^2$

$y = x^2 \quad x = \sqrt{y}$

$$V = \int_0^4 \pi (\sqrt{y})^2 \, dy$$

$V = \int_0^4 \pi (\sqrt{y})^2 dy$ of $y = x^2$

$y = x^2 \quad x = \sqrt{y}$

b)  

$V = \int_0^2 \pi (\text{out rad})^2 - \pi (\text{inn rad})^2 dx$

$\text{out rad} = 4$

inn rad: dist. from graph $y = x^2$ to x-axis

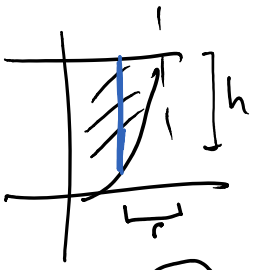
"y" value

$y = x^2$

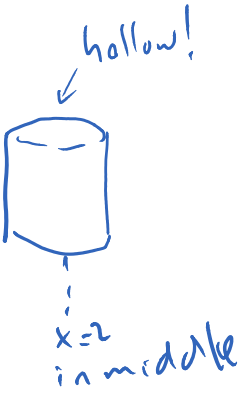
$V = \int_0^2 \pi 4^2 - \pi y^2 dx$

$V = \int_0^2 4^2 \pi - \pi (x^2)^2 dx$

c) About $x = 2$



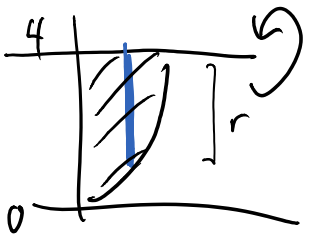
shells



$\int_0^2 2\pi r h \, dx = \int_0^2 2\pi (2-x)(4-y) dx = \int_0^2 2\pi (2-x)(4-x^2) dx$

r = dist between x & 2

h = dist between y & 4

d) 

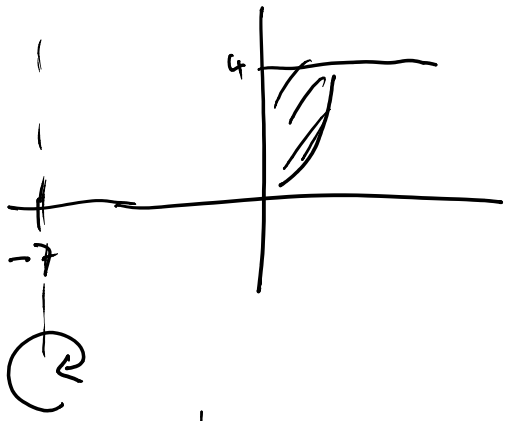
Disks 

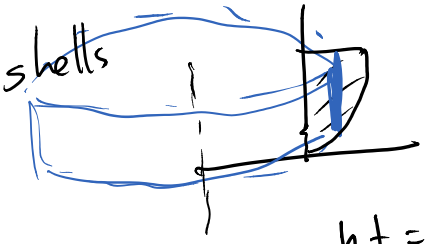
$$V = \int_0^2 \pi r^2 dx = \int_0^2 \pi (4-y)^2 dx$$

r = dist from y to 4

$$= \int_0^2 \pi (4-x^2)^2 dx$$

e) about $x = -7$





shell for each x

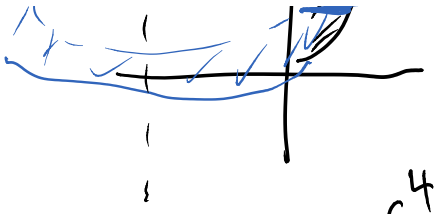
$(4-y)$ = ht = dist from y to 4

$(x-(-7))$ = rad = dist from x to -7

$= (x+7)$

$$\int_0^2 2\pi r h\, dx = \int_0^2 2\pi (x+7)(4-y)\, dx$$

$$= \int_0^2 2\pi (x+7)(4-x^2)\, dx$$



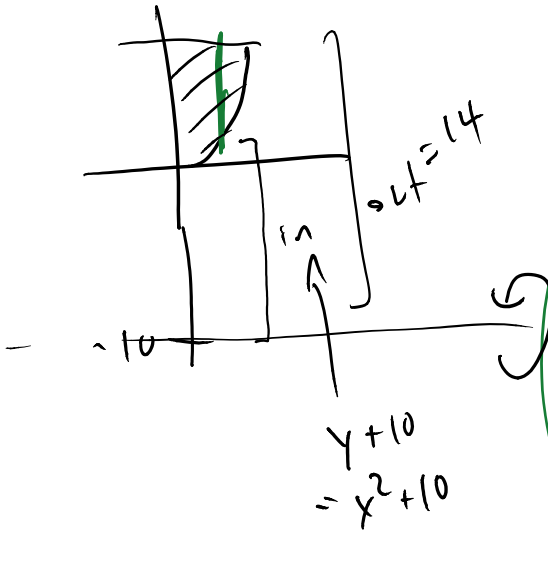
$$= \int_0^2 2\pi(x+7)(4-x^2)\,dx$$

$$\int_0^4 \pi(\text{out rad})^2 - \pi(\text{in rad})^2\,dy = \int_0^4 \pi(\sqrt{y}+7)^2 - \pi 7^2\,dy$$

dist between -7 & $x = \sqrt{y}$ $\;=\; \sqrt{y}+7$

dist between -7 & 0 = y-axis $\;=\; 7$

f)



$$\int_0^2 \pi(\text{out } r)^2 - \pi(\text{in } r^2)\,dx$$

$$= \int_0^2 \pi(14)^2 - \pi(x^2+10)^2\,dx$$