Lecture 1: Introduction and integration review

Tuesday, August 19, 2014 10:52 AM

Integration Review

Recall: $\int f(x)\,dx$ "indefinite integral"

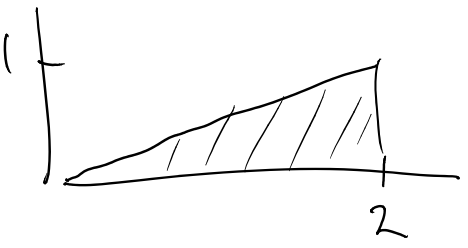
represents general form of an anti-derivative for $f(x)$

$$\int x^2\,dx = \frac{1}{3}x^3 + C$$

$\int_a^b f(x)\,dx$ "definite integral"

signed area between the graph of $f(x)$ & x-axis between $x=a$ & $x=b$

ex



$f(x) = \frac{1}{2}x$

$\int_0^2 \frac{1}{2}x\,dx$

Fundamental Theorem of Calculus:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is any anti-derivative for $f(x)$.

eg. $\frac{1}{4}x^2 = F(x)$ $\quad F'(x) = \frac{1}{2}x$

$$\int_0^2 \frac{1}{2}x\,dx = \frac{1}{4}(2)^2 - \frac{1}{4}(0)^2 = \frac{1}{4}\cdot 4 = 1.$$

Notations $F(b) - F(a) = \left[F(x)\right]_a^b = F(x)\Big|_a^b$

$$= F(x)\Big|_{x=a}^{x=b}$$

Practice

1. $\int_{-2}^{2} (x^3 - 2x + 3)\,dx$

2. $\int \sin x\,dx$

3. $\int \sin 2x\,dx$

4. $\int_{\uparrow}^{1} \sqrt{1-x^2}\,dx$

4. $\int \sin x \, dx$

$\int_{-1}^{1} \; \uparrow$ trick question

5. $\int \frac{4x^3}{(x^4+2)^2} dx$

Reminders $\int x^n dx = \frac{1}{n+1} x^{n+1} + C \qquad n \neq -1$

$\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$

$\int \sin x \, dx = -\cos x + C \qquad \int \cos x \, dx = \sin x + C$