MATH 2250 PRACTICE SHEET FOR FINAL EXAM

1. Use the definition of the derivative to find the derivative of the function
$$f(x) = x^2 + \frac{1}{x}$$

2. Find an equation for the tangent line to the graph of the function
$$f(x) = 3x + \ln x$$
at $x = 1$.
Use this information to approximate $f(1.2)$.

3. Find the derivative of the function
$$f(x) = \frac{xe^x - 1}{\ln x}$$

4. Solve for $\frac{dx}{dt}$ given the equation
$$\ln(x + y) = e^x - t$$

5. Compute the following limit
$$\lim_{x\to 0} \frac{\sin x^2}{\cos x - 1}$$

6. Compute the following limit
$$\lim_{x\to 3} \frac{e^x - e^3}{x - 3}$$

7. Compute the following limit
$$\lim_{x\to 3} \frac{e^x - e^3}{x}$$

8. Find the absolute minimum and maximum values of the function $f(x) = x + \ln x$ on the interval $[1, e]$.

9. Does the following function have an absolute maximum or absolute minimum value on the interval $[\frac{1}{2}, \infty)$?

$$f(x) = x - 7\ln x$$

10. Consider the function $f(x) = \dfrac{3}{1+x^3}$, and suppose that $F(x)$ is an antiderivative for $f(x)$ with $F(0) = 0$.

    Explain why $F(x) = \displaystyle\int_0^x \frac{3}{1+t^3} dt$

11. Two people start walking from the same point, person A walking due north and person B walking due east. After some time, if person A is 40 feet from the starting point and walking at 3 feet per second, and if person B is 30 feet from the starting point and walking at 5 feet per second, how fast is the distance between the two people changing?

12. Compute

$$\int e^x \cos e^x dx$$

13. Compute

$$\int (\sin x)^7 (\cos x) dx$$

14. Compute

$$\int_0^1 x\sqrt{1-x^2}\, dx$$

15. Compute

$$\int_0^1 \sqrt{1-x^2}\, dx$$

hint: this is a trick question

16. Compute

$$\int \tan x\, dx$$

(you shouldn't need to memorize this formula — use u-substitution!)

17. Find two number a and b such that $3a + 4b = 9$ and such that ab is as large as possible.

18. Find all critical values of the following functions $x, x^{-1}, x^2, x^3, x^{2/3}, x^{-2/3}, x + \ln x$.
Which of these critical values represent local minimums and which represent local maximums?

19. Use Riemann Sums with 3 rectangles and using left endpoints to approximate the value of the integral:

$$\int_0^1 \frac{1}{1+x^3}\, dx$$

20. Use Riemann Sums and limits to find the value of the definite integral:

$$\int_2^3 (3x+2)\ dx$$

21. A company would like to design a box (bottom, top and four sides), with square base with a volume of exactly 1000 cubic centimeters. How tall should the box be made so that it uses the least amount of material (surface area)?

22. Suppose that $f(x)$ is defined on $[-3, 3]$ which satisfies the following properties:
- $f(x)$ is increasing on the interval $[-3, 0]$,
- $f(x)$ is decreasing on $[0, 3]$,
- $f(x)$ is concave down on $[-3, 1]$, and
- $f(x)$ is concave up on $[1, 3]$.

Use this information to sketch the graph of $f(x)$.

23. Sketch a graph of a function which is increasing everywhere, concave down for $x < 0$ and concave up for $x > 0$.