

$$1. \int x(x^2+1)^{100} dx = \int u^{100} \left(\frac{1}{2}\right) du$$

$$u = x^2 + 1$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$= \frac{1}{2} \frac{1}{101} u^{101} + C$$

$$= \left(\frac{1}{2}\right) \left(\frac{1}{101}\right) (x^2+1)^{101} + C$$

$$2. \int \frac{\sin x}{\cos^3 x} dx$$

$$4. \int \sec^2 x \tan x dx = \int \frac{\sin x}{\cos^3 x} dx = - \int \frac{dy}{u^2} = - \int u^{-3} dy$$

$$u = \cos x$$

$$du = -\sin x dx$$

$$= \frac{1}{2} u^{-2} + C$$

$$= \frac{1}{2} (\cos^2 x)^{-1} + C$$

$$= \int \sec x (\sec x \tan x) dx$$

$$3. \int \tan x dx = \int \frac{\sin x}{\cos x} dx = - \int \frac{du}{u} = -\ln |u| + C$$

$$u = \cos x$$

$$du = -\sin x dx$$

$$-du = \sin x dx$$

$$= \boxed{-\ln |\cos x| + C}$$

$$= \ln |\cos x|^{-1} + C$$

$$= \ln \frac{1}{|\cos x|} + C$$

$$= \boxed{\ln |\sec x| + C}$$

$$5. \int \sin^7 x dx = \int \sin^6 x \sin x dx = \int (\sin^2 x)^3 \sin x dx$$

$$= \int (1 - \cos^2 x)^3 \sin x dx$$

$$u = \cos x$$

$$du = -\sin x dx$$

$$\begin{aligned} &= -\int (1-u^2)^3 du = -\int (1-3u^2+3u^4-u^6) du \\ &\quad \text{(foil, foil, foil)} \\ &= -\left(u - u^3 + \frac{3}{5}u^5 - \frac{1}{7}u^7\right) + C \\ &= -u + u^3 - \frac{3}{5}u^5 + \frac{1}{7}u^7 + C \\ &= -\cos x + \cos^3 x - \frac{3}{5}\cos^5 x + \frac{1}{7}\cos^7 x + C \end{aligned}$$