

## Curve sketching (towards optimization)

Basic tool: sign chart (where is a fun pos or neg)

to make sign chart: idea is that a fun changes sign only by  
1) passing through 0 or  
2) being discontinuous

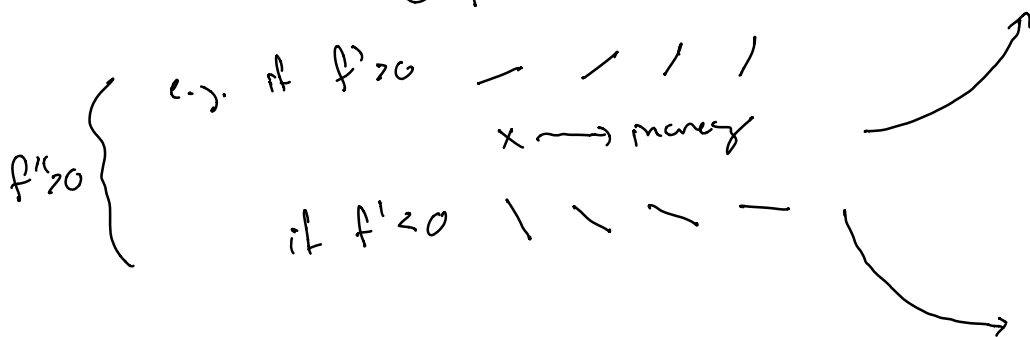
Can do sign charts for

$f(x)$  when is  $f(x)$  above or below  $x$ -axis  
 $f'(x)$  when is  $f(x)$  inc./dec.  
 $f''(x)$

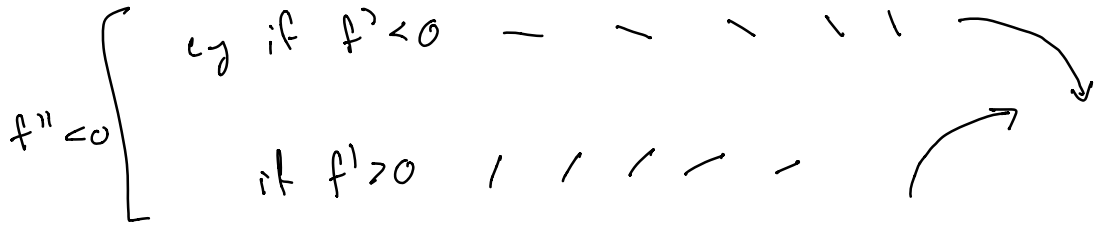
Second derivative  $f''(x)$  derivative of derivative

tells about rate of change of the derivative

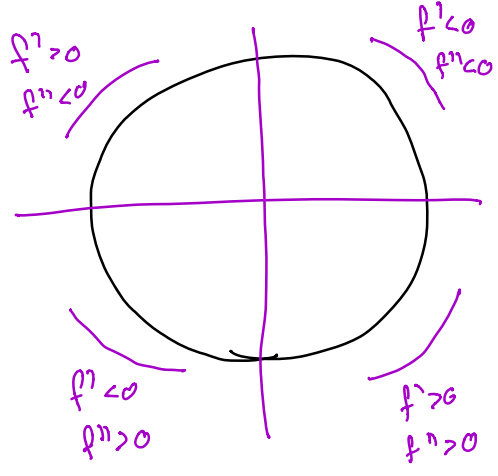
ex:  $f'' > 0$  means as  $x$  increases,  $f'$  increases  
(slopes get more positive)



if  $f'' < 0$  means as  $x$  increases,  $f'$  decreases



	$f'' > 0$	$f'' < 0$
$f' > 0$		
$f' < 0$		

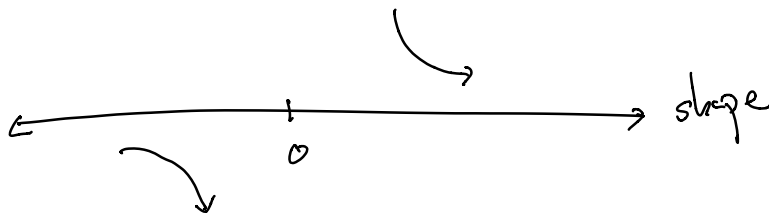
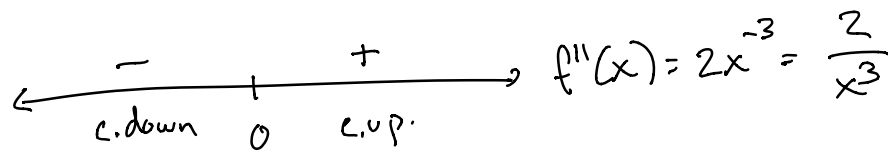
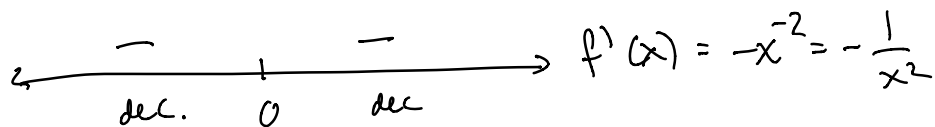
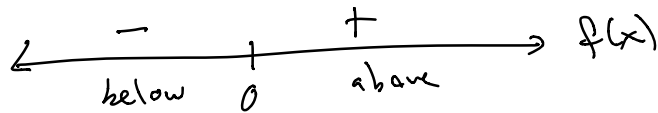


$f'' > 0$            concave up      c.up

$f'' < 0$            concave down      c.down

Example  $f(x) = \frac{1}{x} = x^{-1}$

sign chart for  $f(x)$



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example  $f(x) = 2x^3 - 3x^2 - 12x$

←  $f(x)$  →

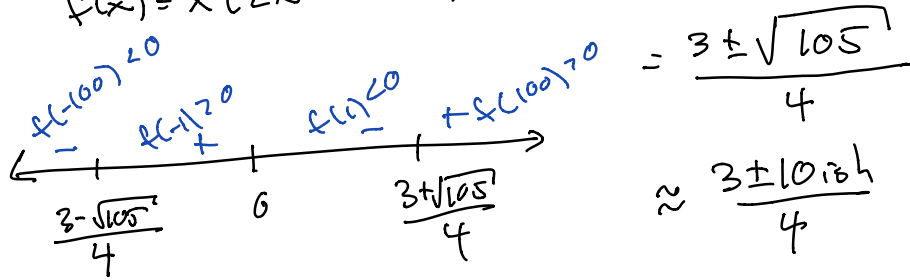
$$f(x) = 0 = 2x^3 - 3x^2 - 12x$$

$$= x(2x^2 - 3x - 12)$$

$$\text{so } x = 0 \text{ or } 2x^2 - 3x - 12 = 0$$

$$2x^2 - 3x - 12 = 0 \Rightarrow x = \frac{3 \pm \sqrt{9 + 4(2)(12)}}{2(2)}$$

$$f(x) = x(2x^2 - 3x - 12)$$

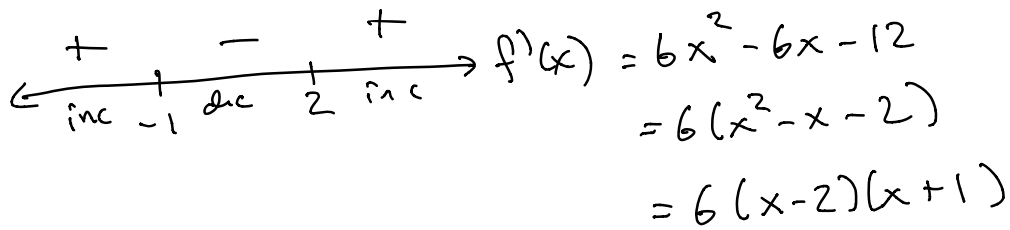


$$= \frac{3 \pm \sqrt{105}}{4}$$

$$\approx \frac{3 \pm 10.25}{4}$$

$$-\frac{7}{4} \text{ or } \frac{13}{4}$$

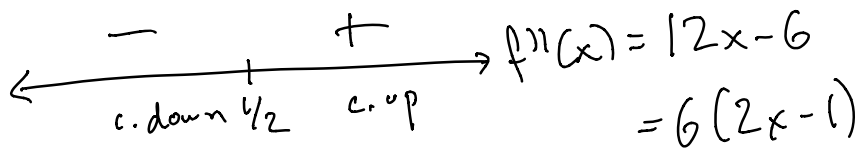
$\approx -2.75$  or  $3.25$ .



$$f'(x) = 6x^2 - 6x - 12$$

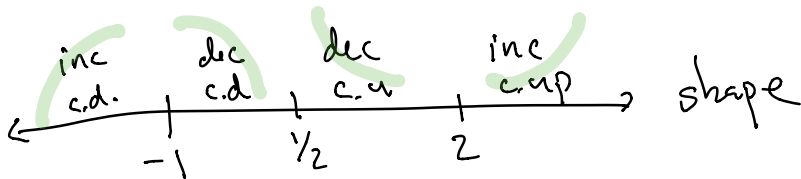
$$= 6(x^2 - x - 2)$$

$$= 6(x-2)(x+1)$$



$$f''(x) = 12x - 6$$

$$= 6(2x - 1)$$



shape



