Lecture 10: More chain rule

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$f(x) = e^{\sin x}$ $\quad u = \sin(x)$ $\quad \dfrac{du}{dx} = \cos x$

$f(u) = e^u$ $\quad \dfrac{df}{dx} = \dfrac{df}{du}\dfrac{du}{dx}$

$\dfrac{df}{du} = e^u$ $\quad f'(x) = e^u \cdot \cos x$

$= e^{\sin x} \cdot \cos x$

$\dfrac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x) = e^{\sin x} \cdot \cos x$

warning notation changing!

$f(g(x)) = e^{\sin x}$

$g(x) = \sin x$ $\quad f(x) = e^x$

$g'(x) = \cos x$ $\quad f'(x) = e^x$

Practice

1. $\dfrac{d}{dx} \sin^2 x = \dfrac{d}{dx}(\sin x)^2 = f'(g(x)) \cdot g'(x) = 2(\sin x) \cdot \cos x$

$f(g(x))$

$f(x) = x^2$ $\quad f'(x) = 2x$

$g(x) = \sin x$ $\quad g'(x) = \cos x$

2. $\dfrac{d}{dx} \sin(\sin x) = f'(g(x)) \cdot g'(x) = \cos(\sin x) \cdot \cos x$

$(x) = \sin x$

2. $\frac{d}{dx}\sin(\sin x)$

$f(x) = \sin x \qquad g(x) = \sin x$

$f'(x) = \cos x \qquad g'(x) = \cos x$

$2\frac{1}{2}.\quad \frac{d}{dx}\frac{1}{x^6} = \frac{d}{dx}x^{-6} = -6x^{-7}$

3. $\frac{d}{dx}\frac{1}{\sin^6 e^x} = \frac{d}{dx}\frac{1}{(\sin(e^x))^6} = \frac{d}{dx}(\sin(e^x))^{-6}$

$f(x) = x^{-6} \qquad g(x) = \sin(e^x)$

$f'(x) = -6x^{-7} \qquad g'(x) = \ldots$

$-6(\sin e^x)^{-7}\cdot g'(x) = -6(\sin e^x)^{-7}\frac{d}{dx}(\sin(e^x))$

$f(x) = \sin x \qquad g(x) = e^x$

$f'(x) = \cos x \qquad g'(x) = e^x$

$-6(\sin e^x)^{-7}\cos(e^x)\cdot e^x.$

More practice

1. $\frac{d}{dx}\sin(\cos(\tan x))$ $\quad f(x) = \sin x$, $g(x) = \cos(\tan x)$

$1\frac{1}{2}.\quad \frac{d}{dx}e^{2x}$ $\quad f(x) = e^x$, $g(x) = 2x$

2. $\frac{d}{dx}e^{(x^2+2x+1)}$ $\quad f(x) = e^x$, $g(x) = x^2+2x+1$

3. $\frac{d}{dx}\sin^9 x$

4. $\frac{d}{dx}\sin^9(2x+1)$ $\quad f(x) = x^9$, $g(x) = \sin(2x+1)$

$e^{\ln x} = x$ take derivative of both

$f(x) = x^{a}$ $\rightarrow$ ax
$g(x) = \sin x$

5. notice $e^{\ln x} = x$ take derivative of both sides to solve for $\frac{d}{dx} \ln x$.